A non-comparison oblivious sort and its application to private k-NN

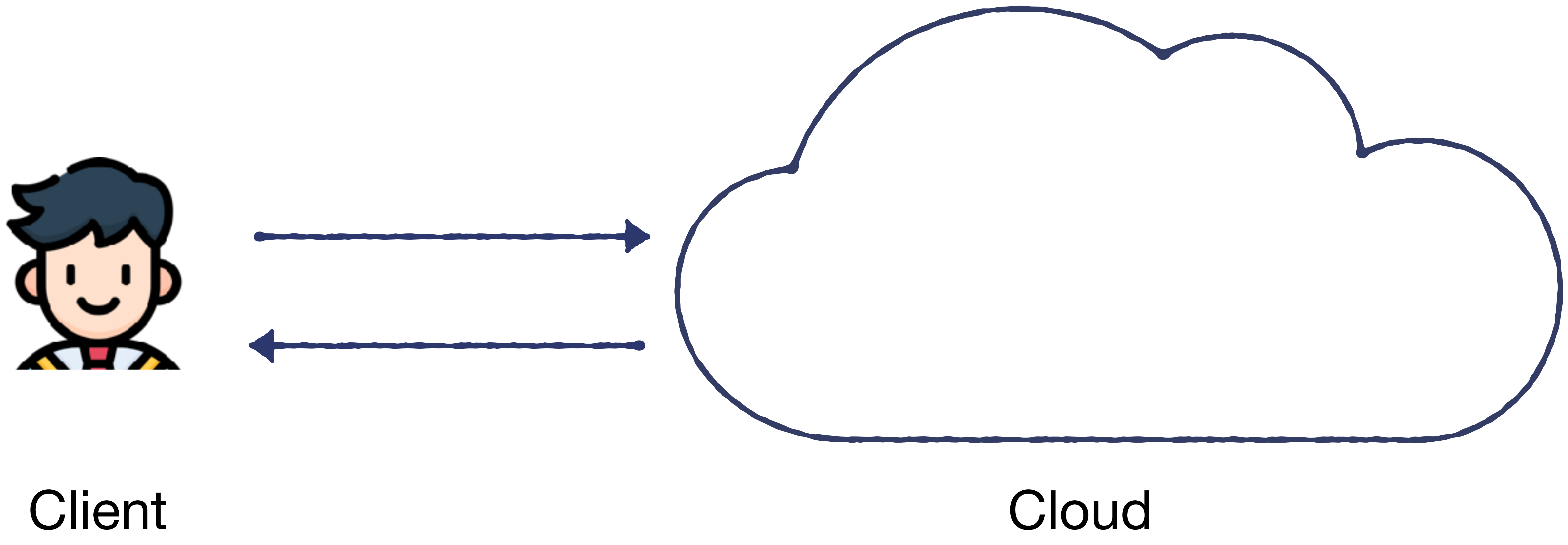
Sofiane Azogagh, Marc-Olivier Killijian and Félix Larose-Gervais

PETS'25 - July 17th 2025

Introduction and context

Context and security model

Outsourcing the computation



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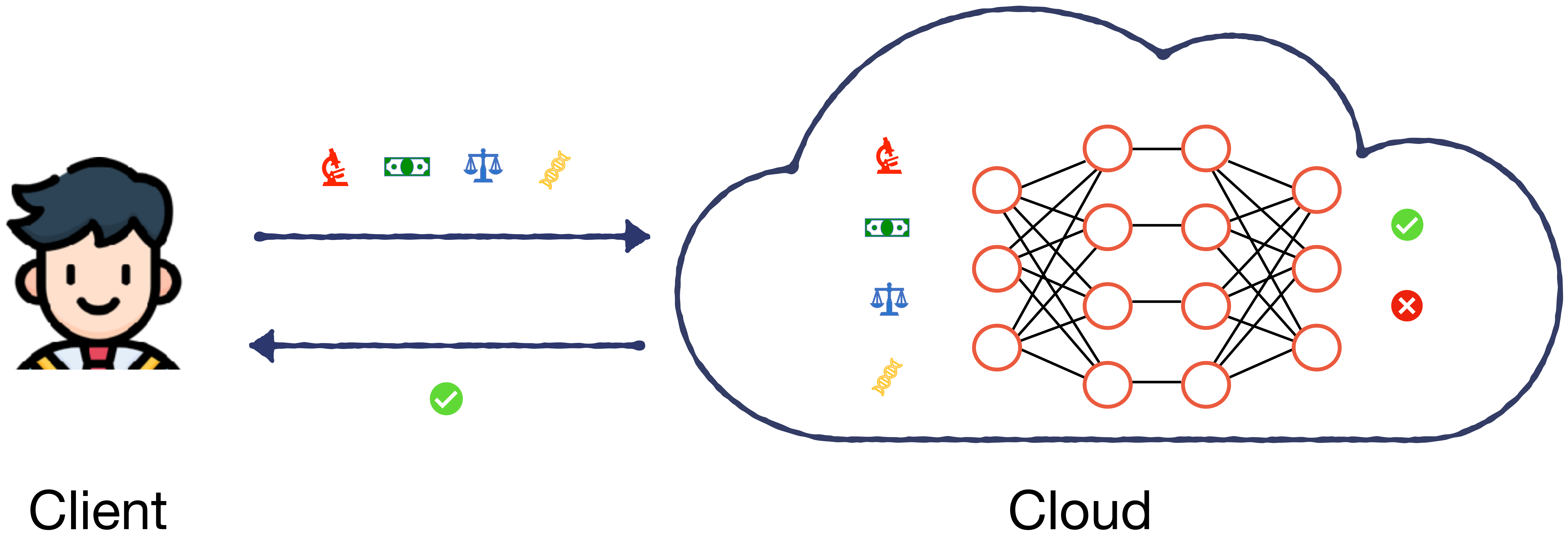

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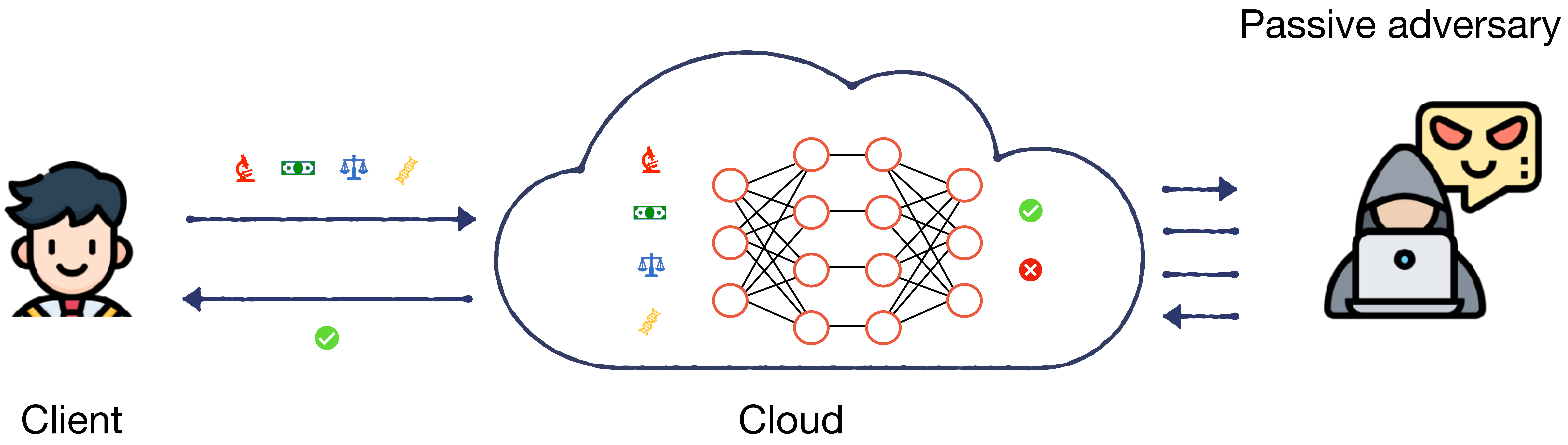

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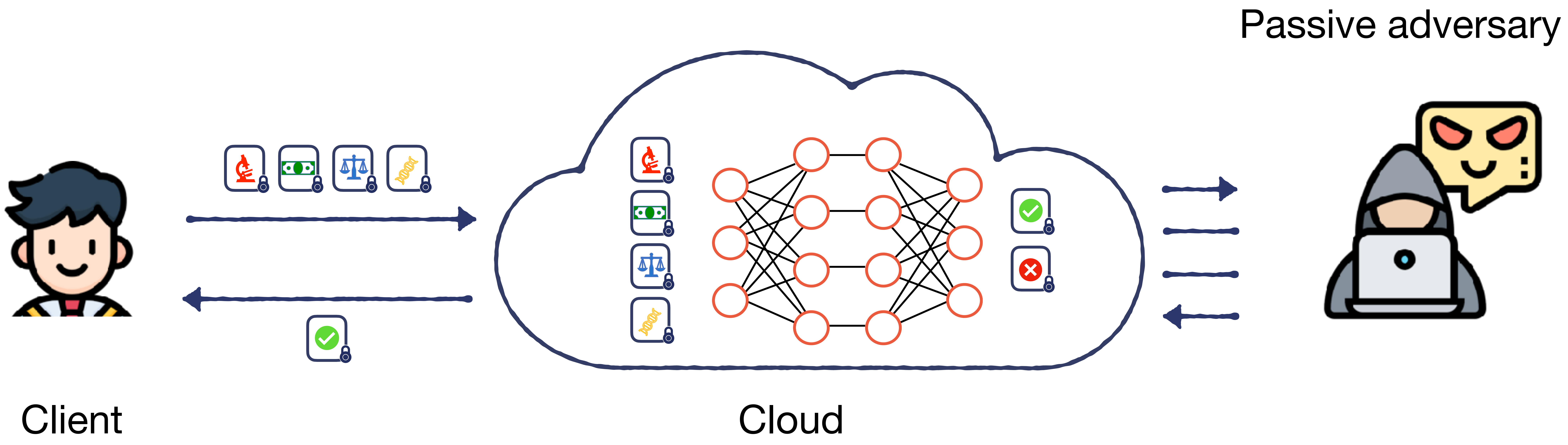

Oblivious algorithm

Just a definition

```
if a > b:
    max = a
else:
    max = b
```

Non oblivious

```
s = (a>b)
max = a*s + b*(1-s)
```

Oblivious

Oblivious algorithms are algorithms whose **access patterns** (e.g., which memory addresses they touch) and **control flow** (e.g., which branch they take) are **independent of the input data values.**

Fully Homomorphic Encryption

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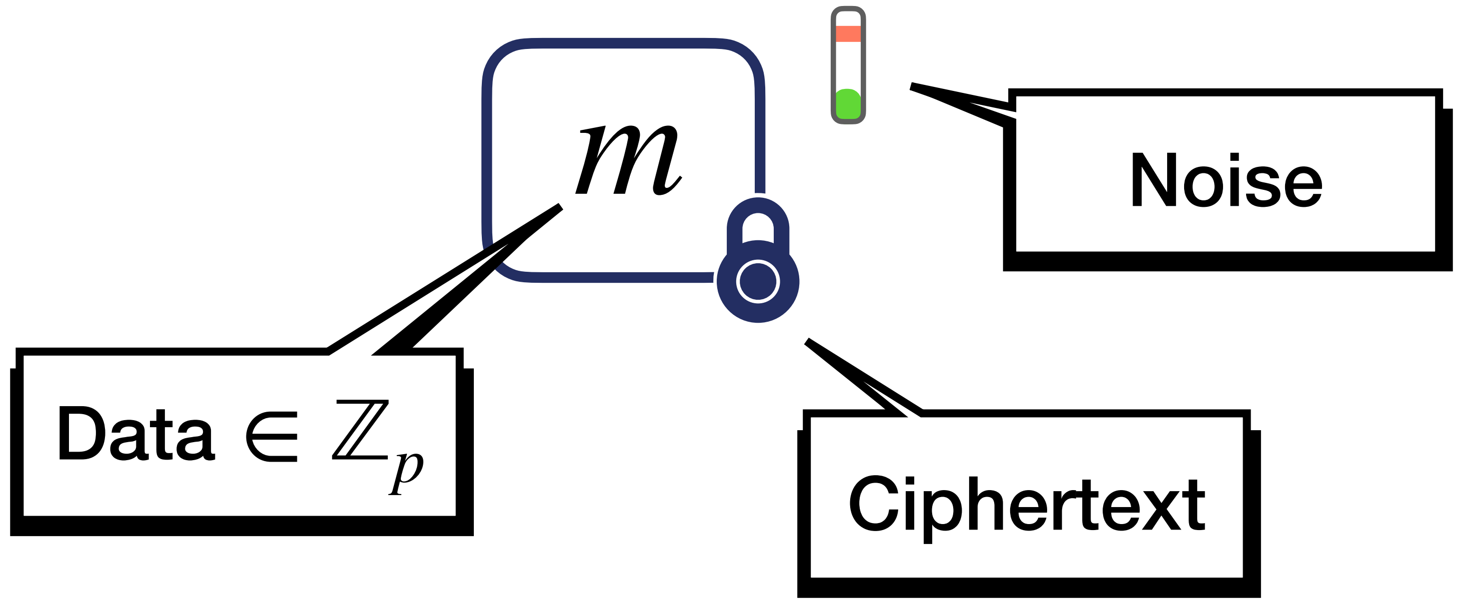

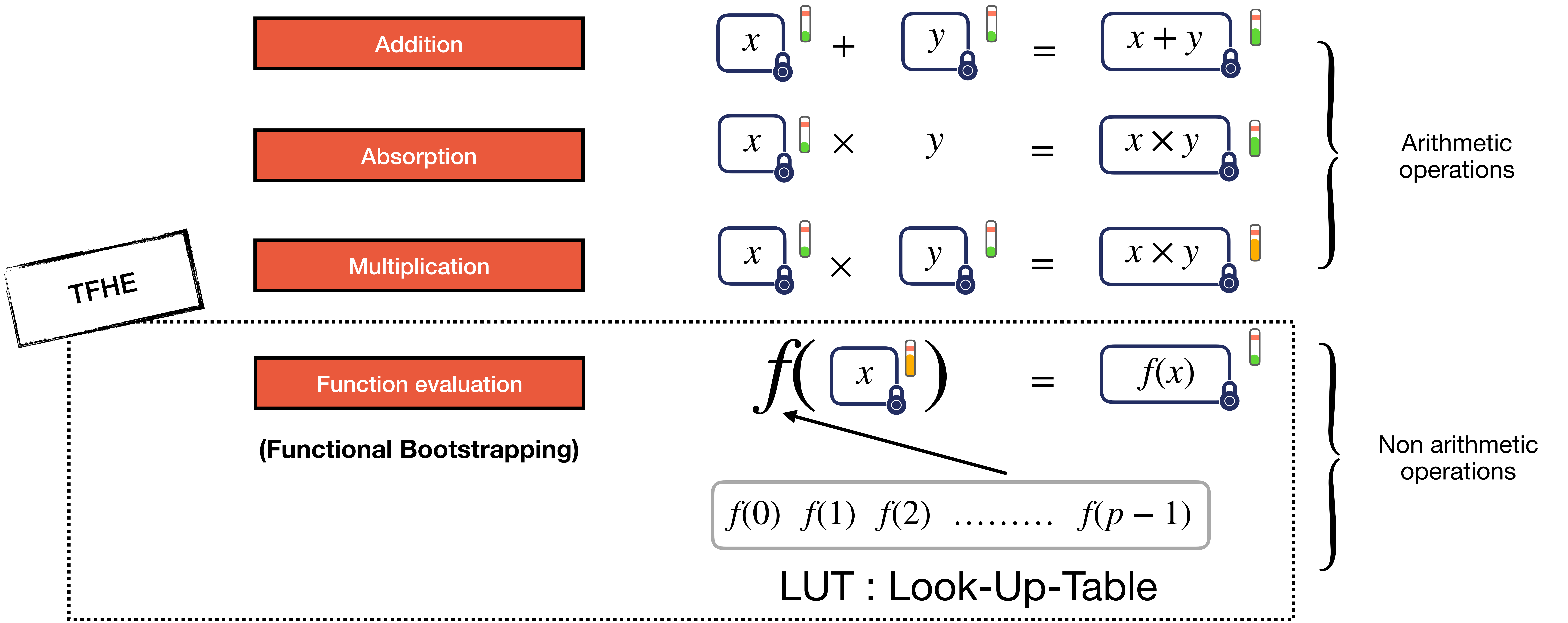

Chillotti et al. : TFHE: Fast Fully Homomorphic Encryption Over the Torus. J Cryptol 33, 34–91 (2020)

TFHE cryptosystem

Different types of ciphertexts

TFHE cryptosystem

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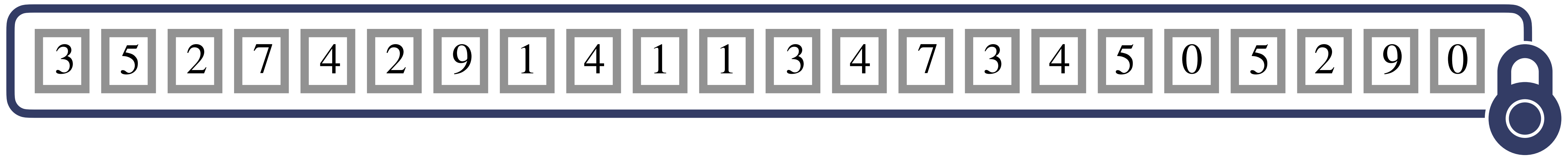
LWE ciphertext

TFHE cryptosystem
Different types of ciphertexts

LWE ciphertext

RLWE ciphertext



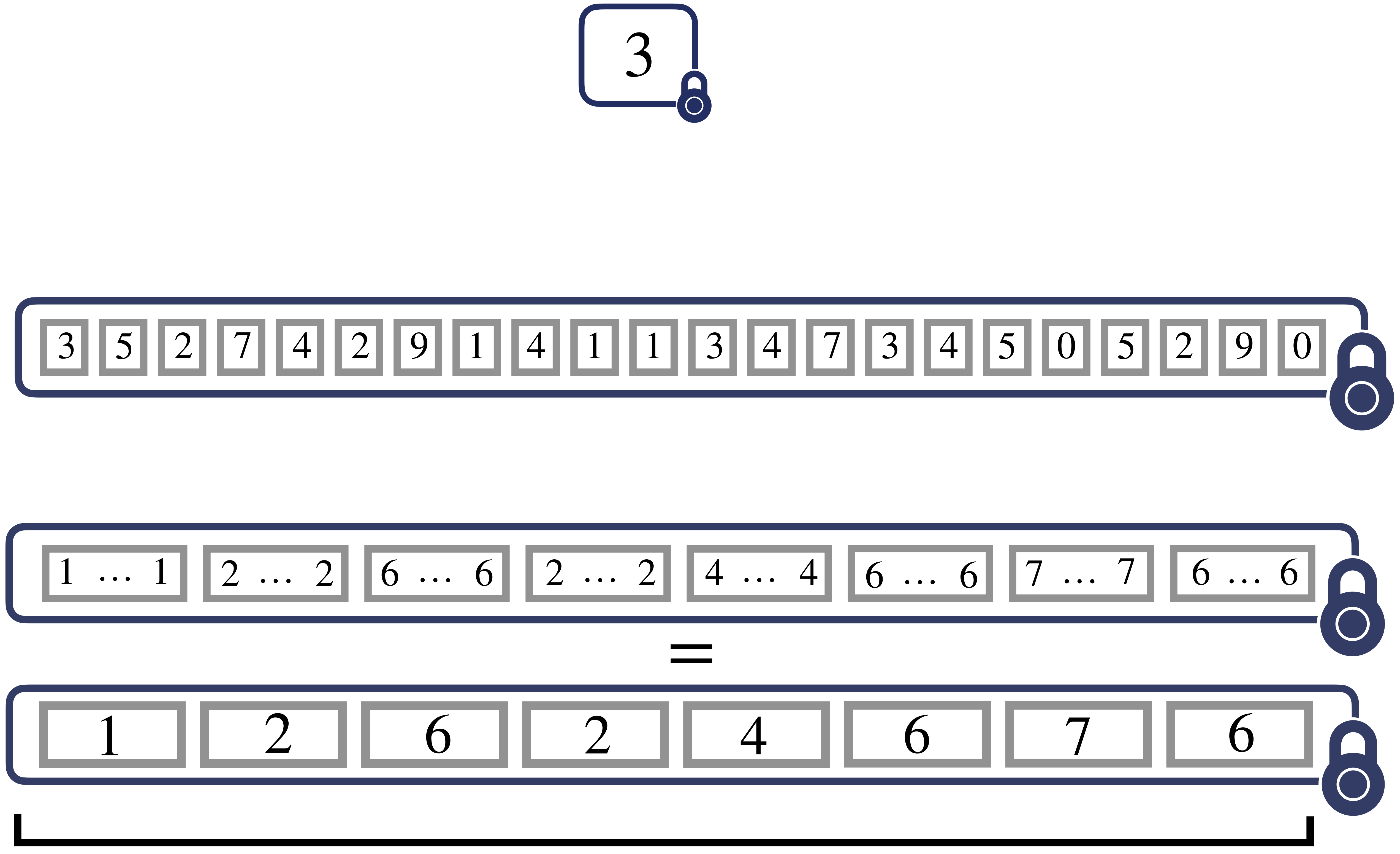
TFHE cryptosystem

Different types of ciphertexts

LWE ciphertext

RLWE ciphertext

LUT ciphertext



p elements of $\mathbb{Z}_p$

TFHE cryptosystem

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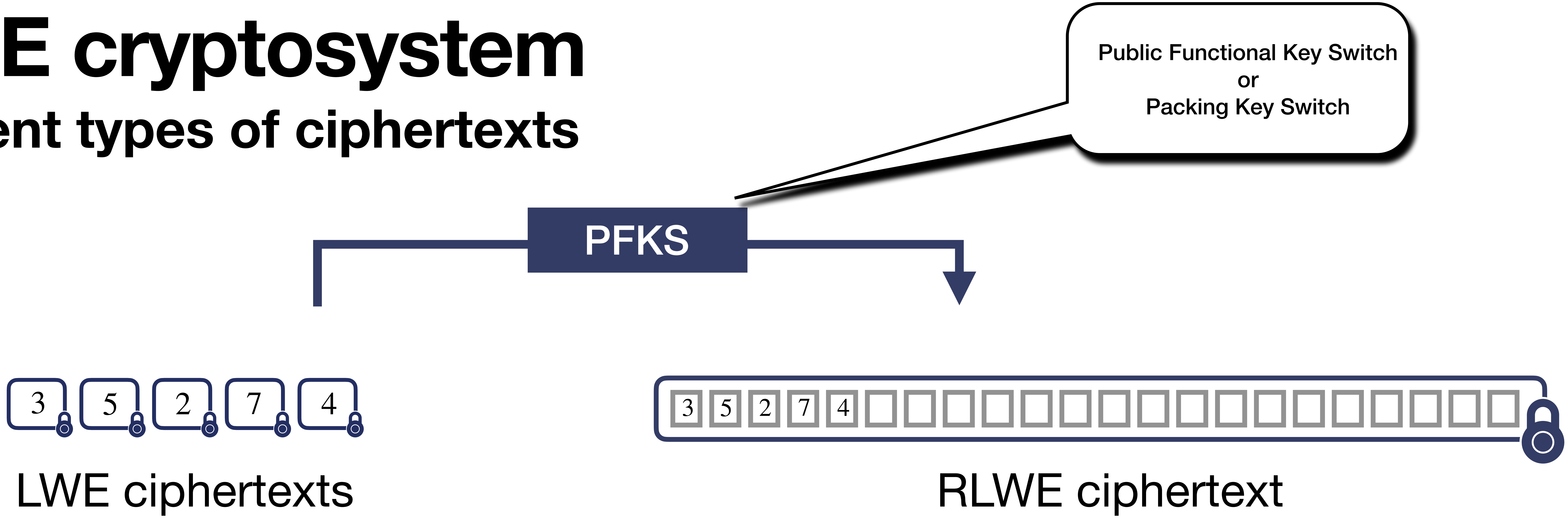

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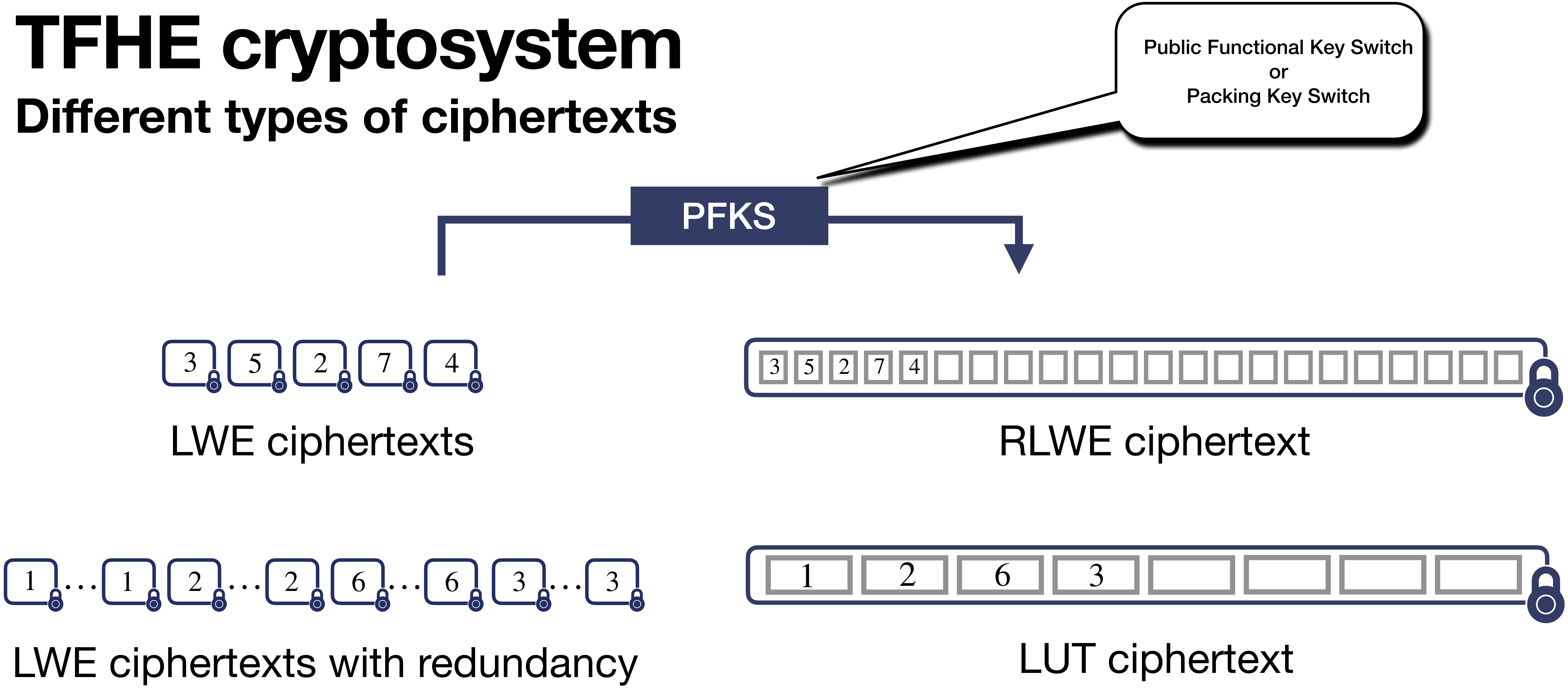
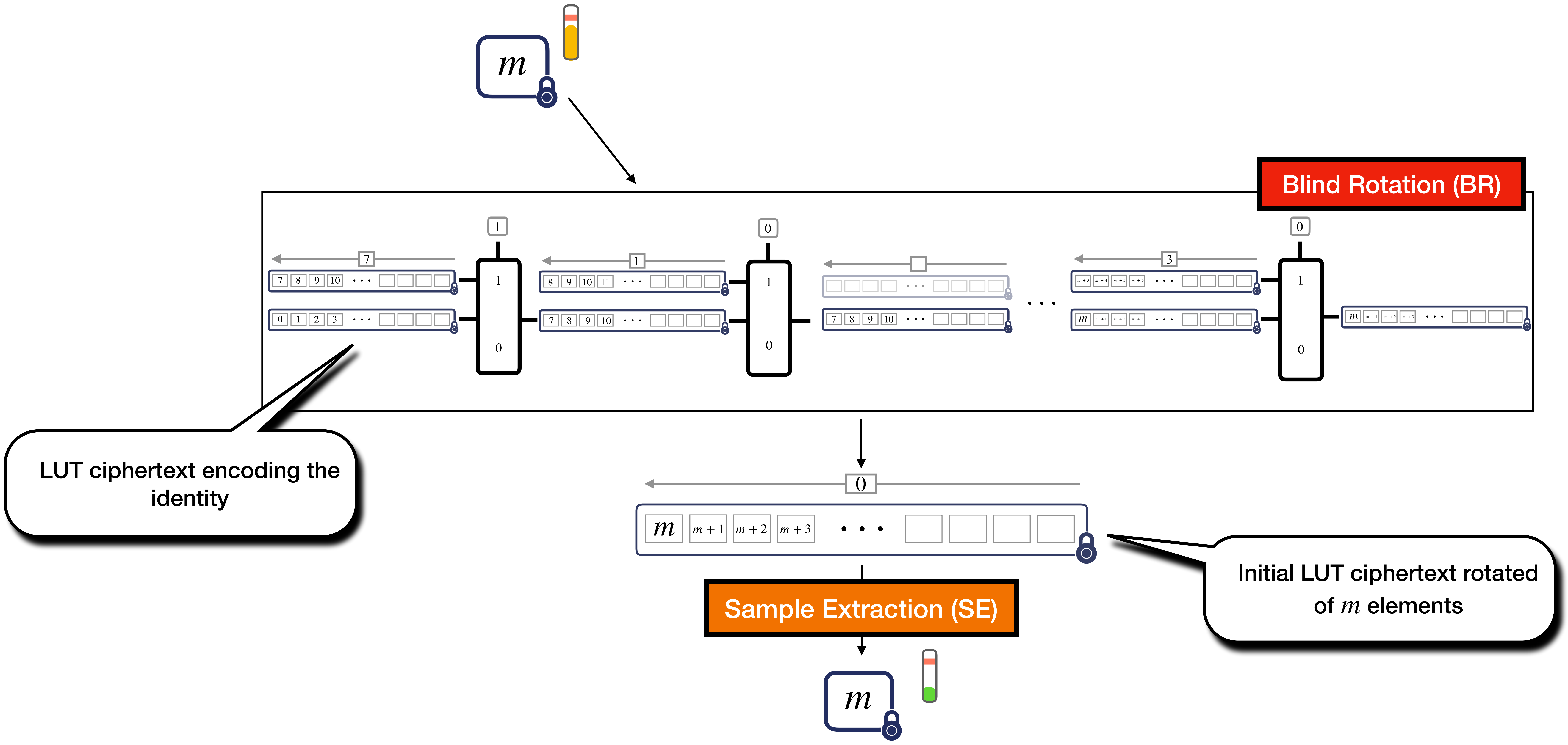


TFHE cryptosystem

High level view of the bootstrapping procedure



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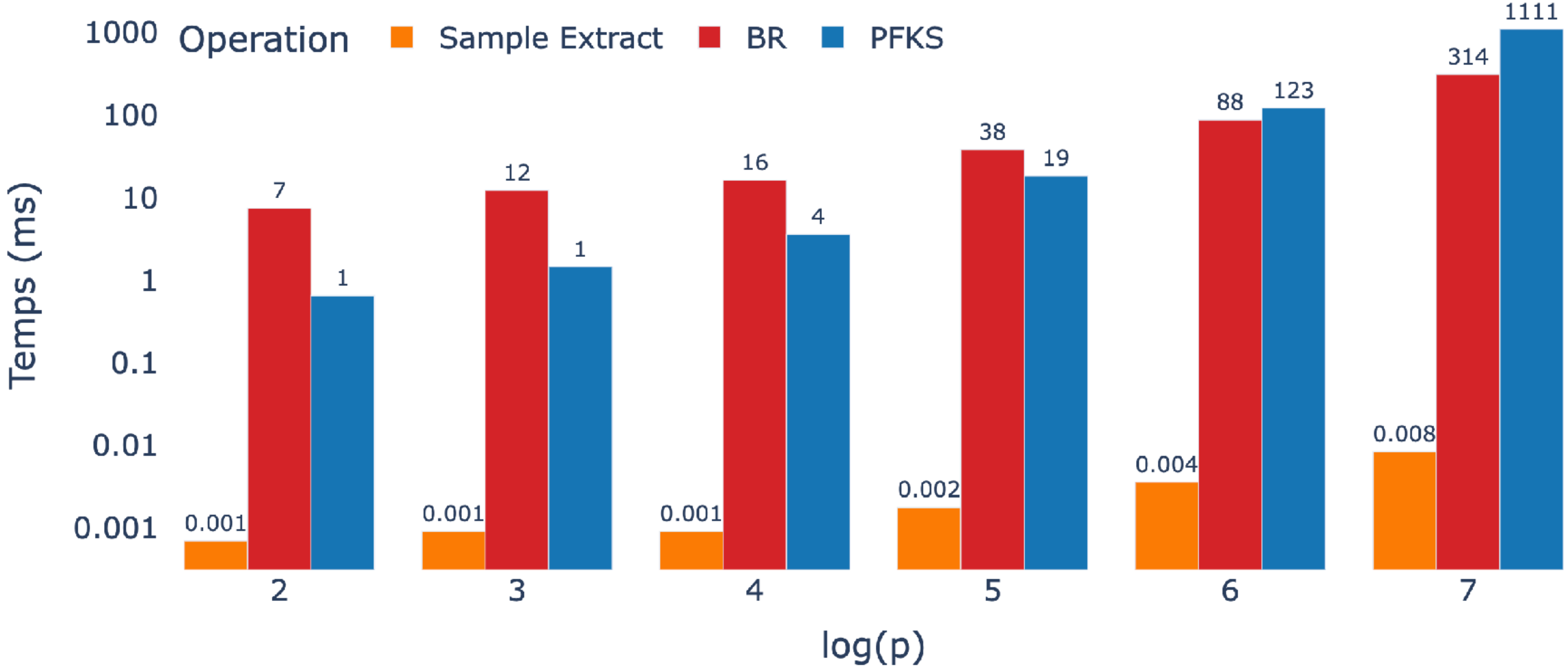


RevoLUT : Rust Efficient Versatile Oblivious Look-Up-Table

ZAMA
TFHE-rs

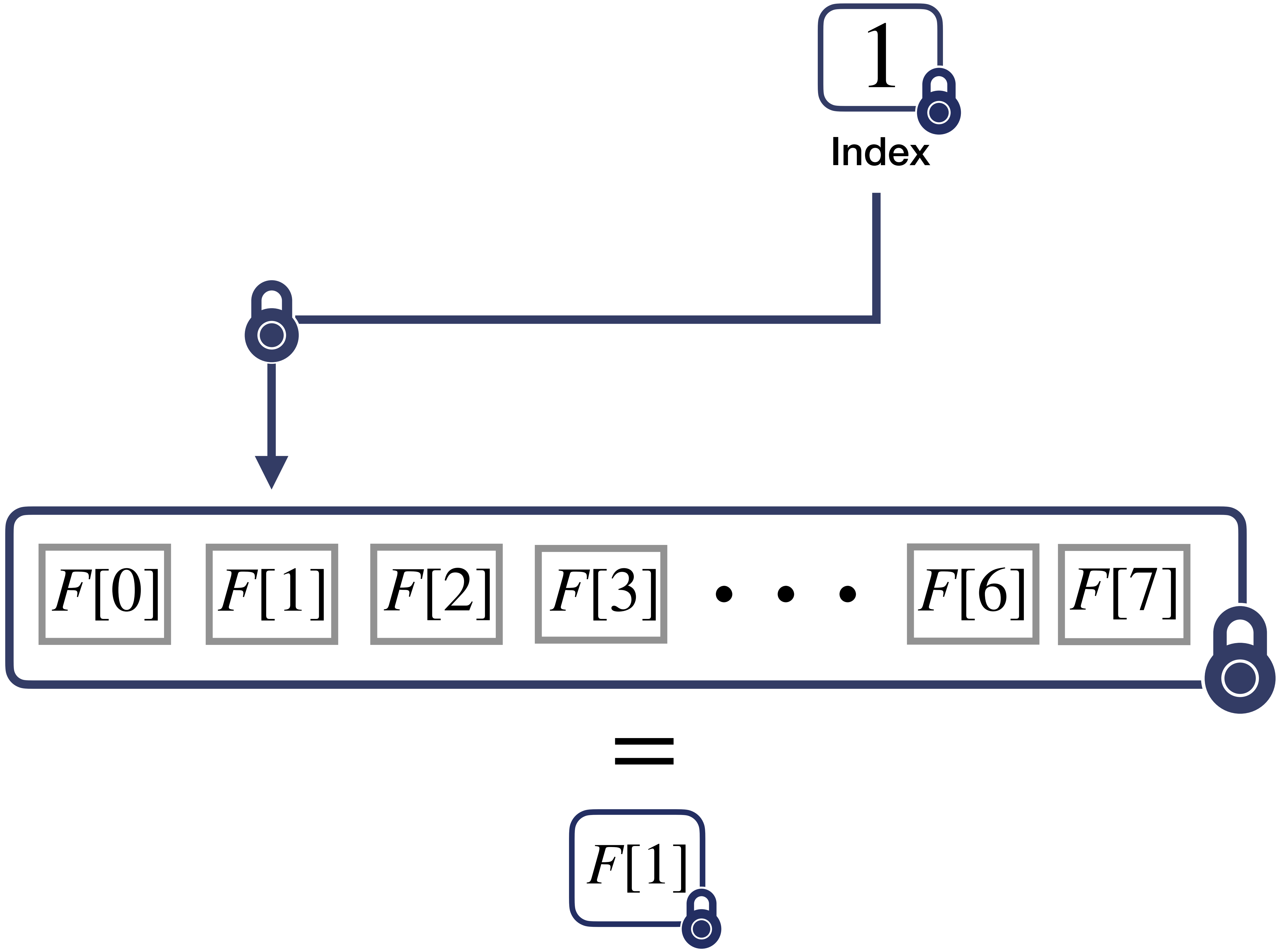
RevoLUT library

sofianeazogagh/revoLUT

How to blindly read in an encrypted array ?

Blind Array Access

$$t_{BAA} = 1 \times \text{BR}$$



RevoLUT library

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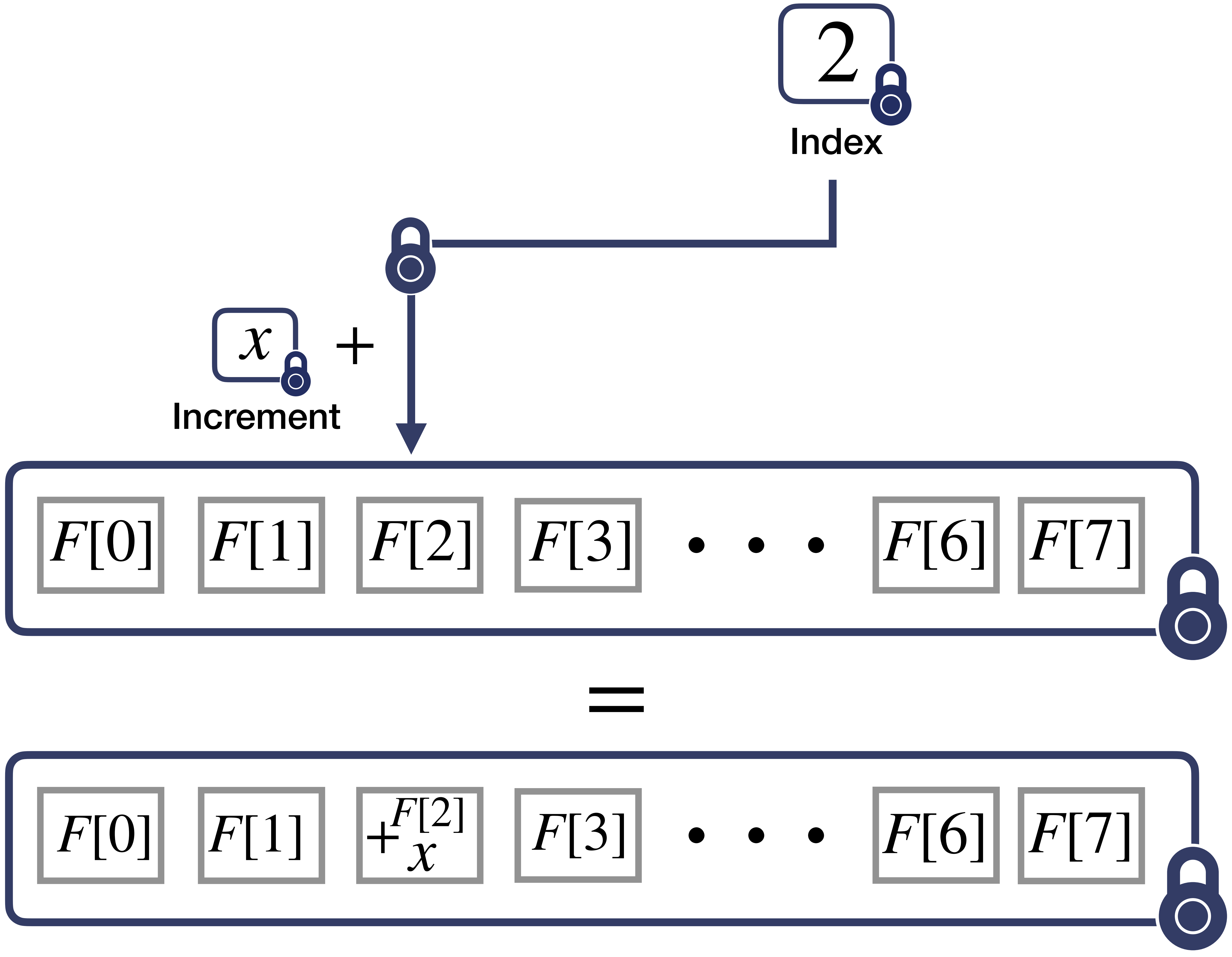
How to blindly write in an encrypted array ?

Blind Array Add

$$t_{BAAdd} = 1 \times \text{BR} + 1 \times \text{PFKS}$$



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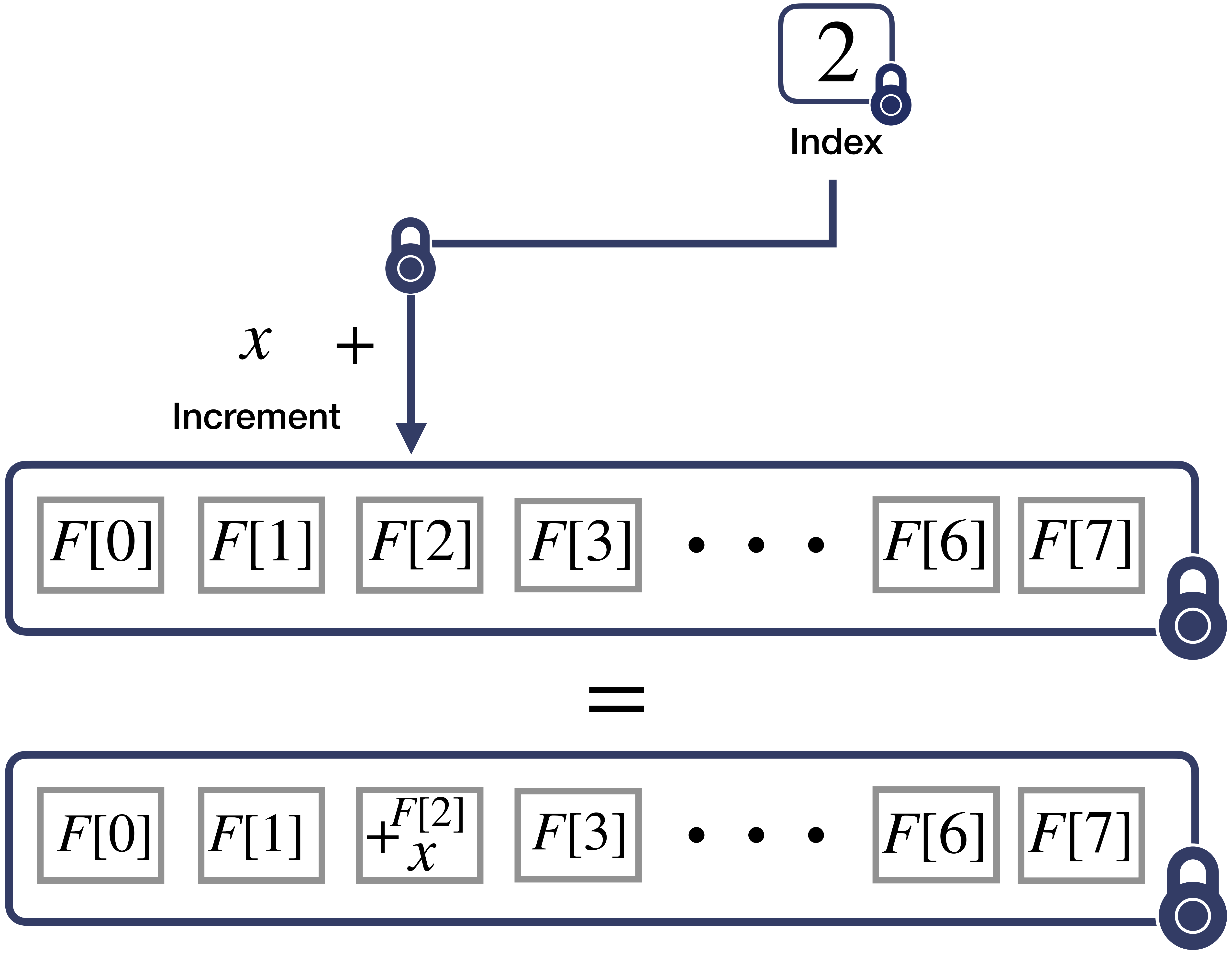

Blind Counting Sort (BCS)

Counting Sort

How it works in clear ?

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$$\mathcal{O}(n + p)$$

number of items

items $\in \mathbb{Z}_p$

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$n = 8$ $\quad$ $p = 6$

3	2	5	2	4	1	5	1

Unsorted array A

Counting Sort

How it works in clear ?

$$\mathcal{O}(n + p)$$

number of items

items $\in \mathbb{Z}_p$

1) Count the occurrences : $\mathcal{O}(n)$

0	1	2	3	4	5
0	2	2	1	1	2

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2) Build the prefix sum : $\mathcal{O}(p)$

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0	2	4	5	6	8

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<u>Destination range</u> :

- $[0,2[$ should be filled of 1s
- $[2,4[$ should be filled of 2s
- $[4,5[$ should be filled of 3s
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3) Rebuild the sorted array : $\mathcal{O}(n)$

0	1	2	3	4	5	6	7

Sorted array

Counting Sort

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$n = 8$ $p = 6$

3	2	5	2	4	1	5	1

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0	1	2	3	4	5	6	7
1	1	2	2	3	4	5	5

Sorted array

Counting Sort

The challenge of bringing it to the encrypted world

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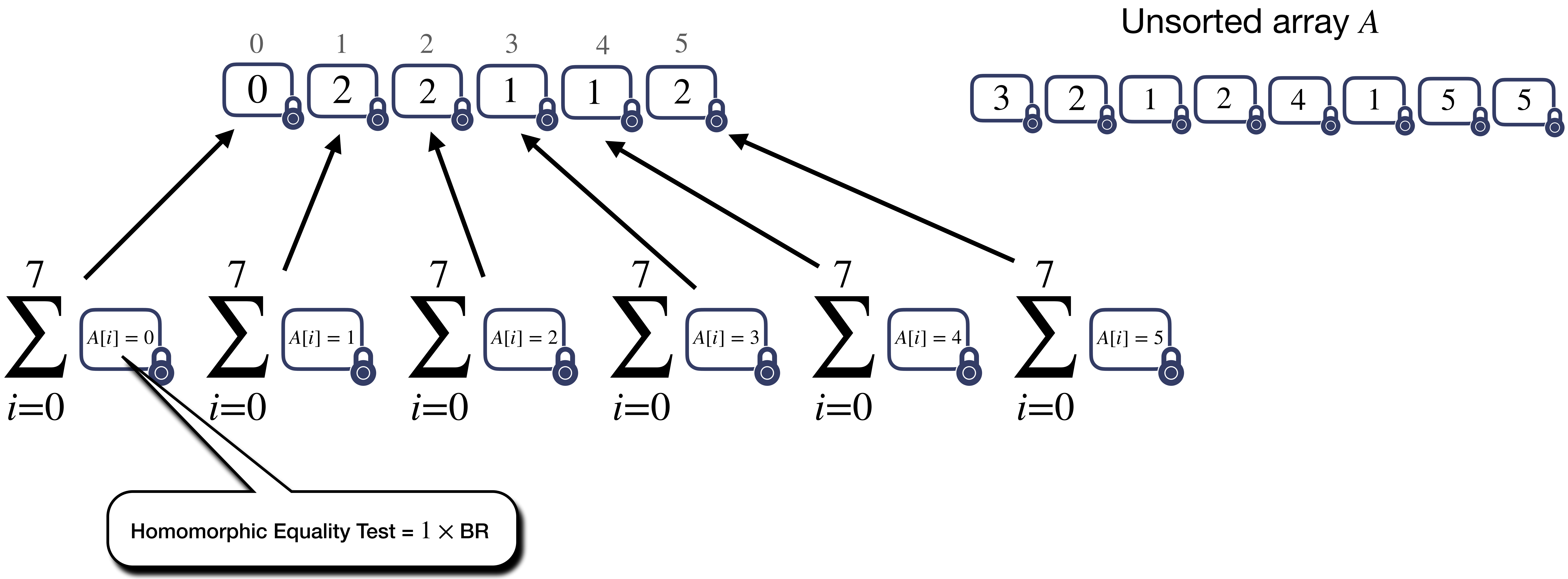
Count the occurences : $\mathcal{O}(n^2)$



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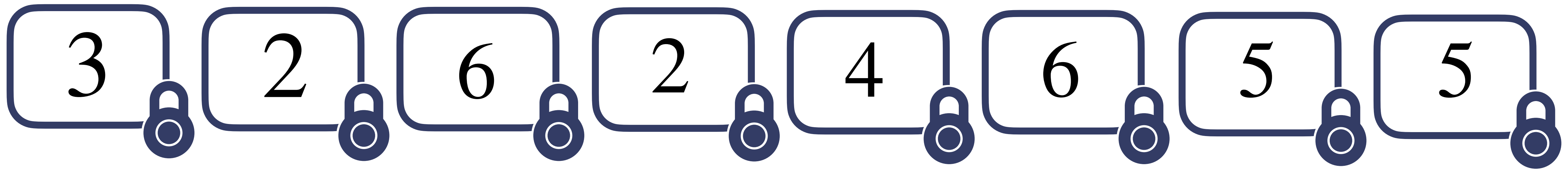

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$\mathcal{O}(n)$

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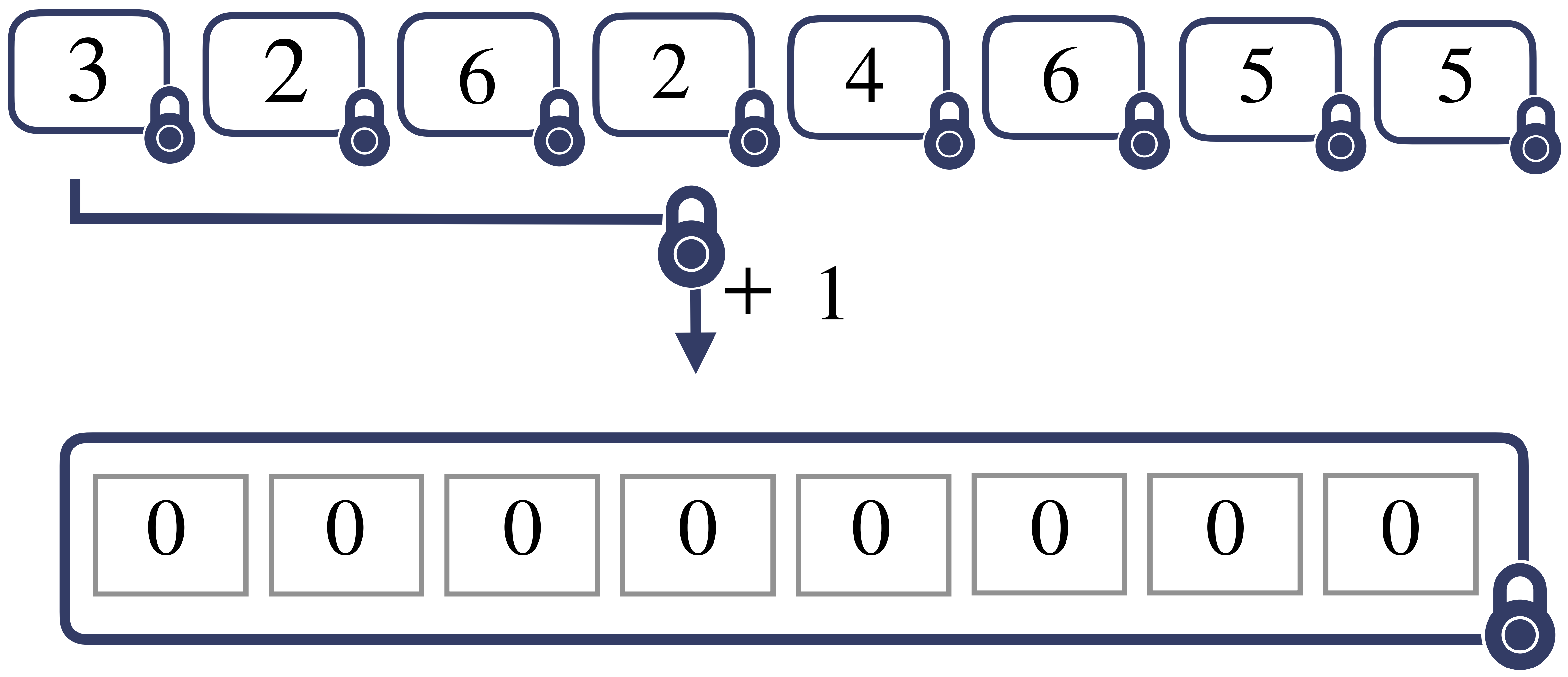

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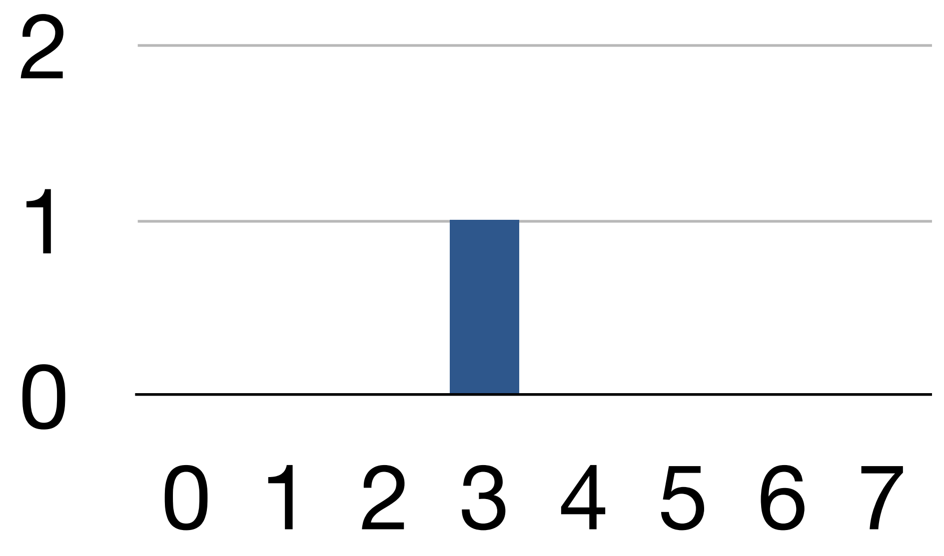

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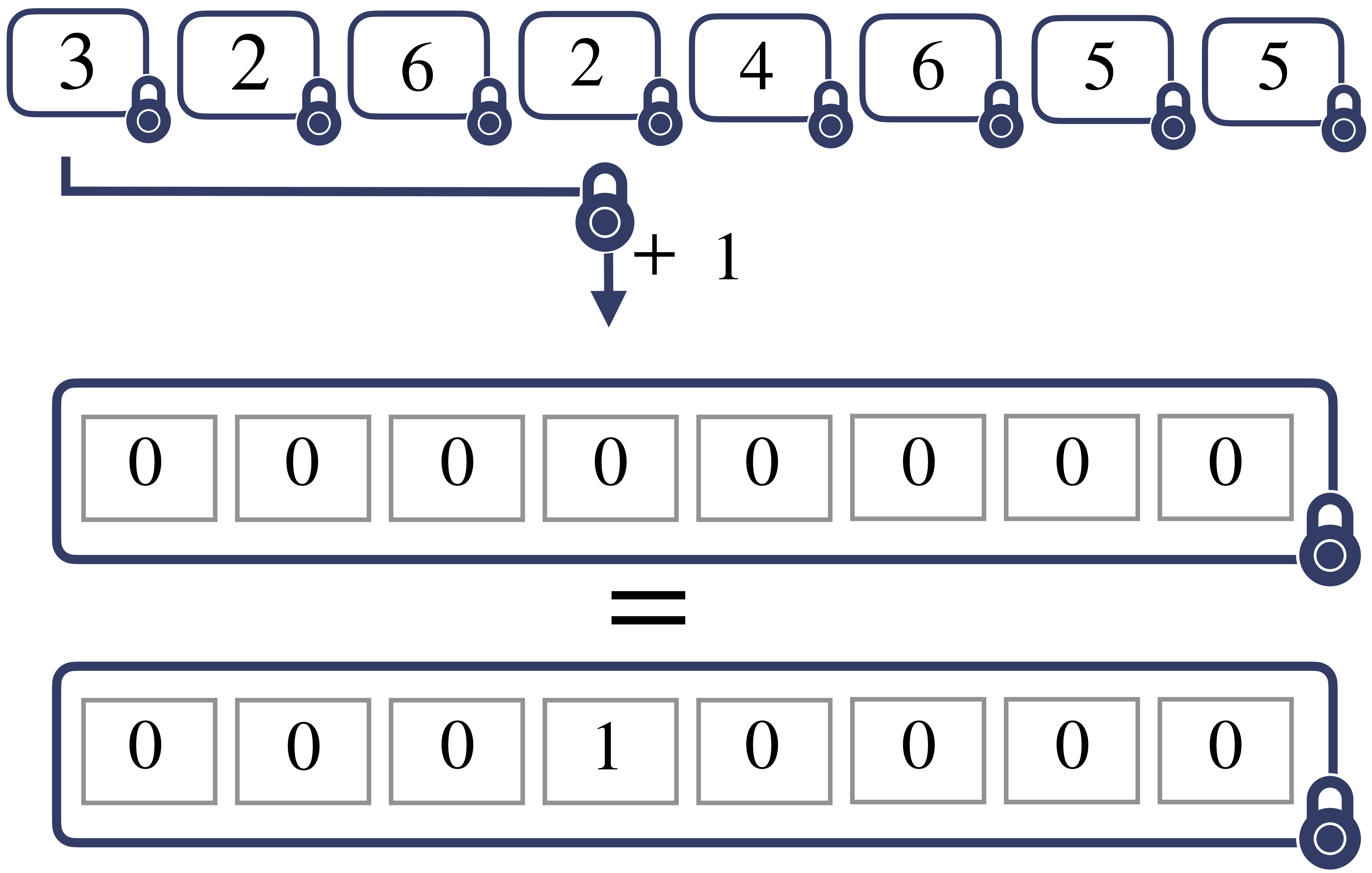

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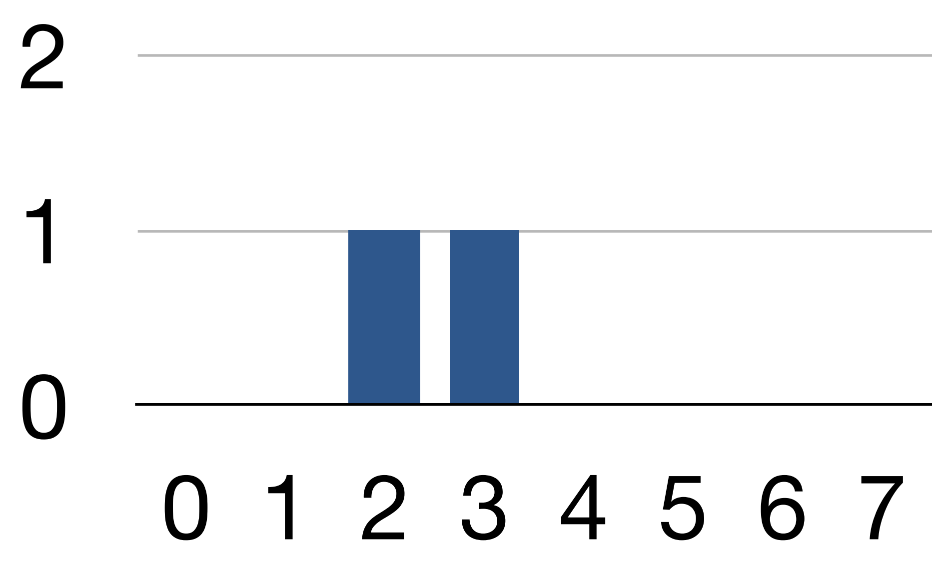

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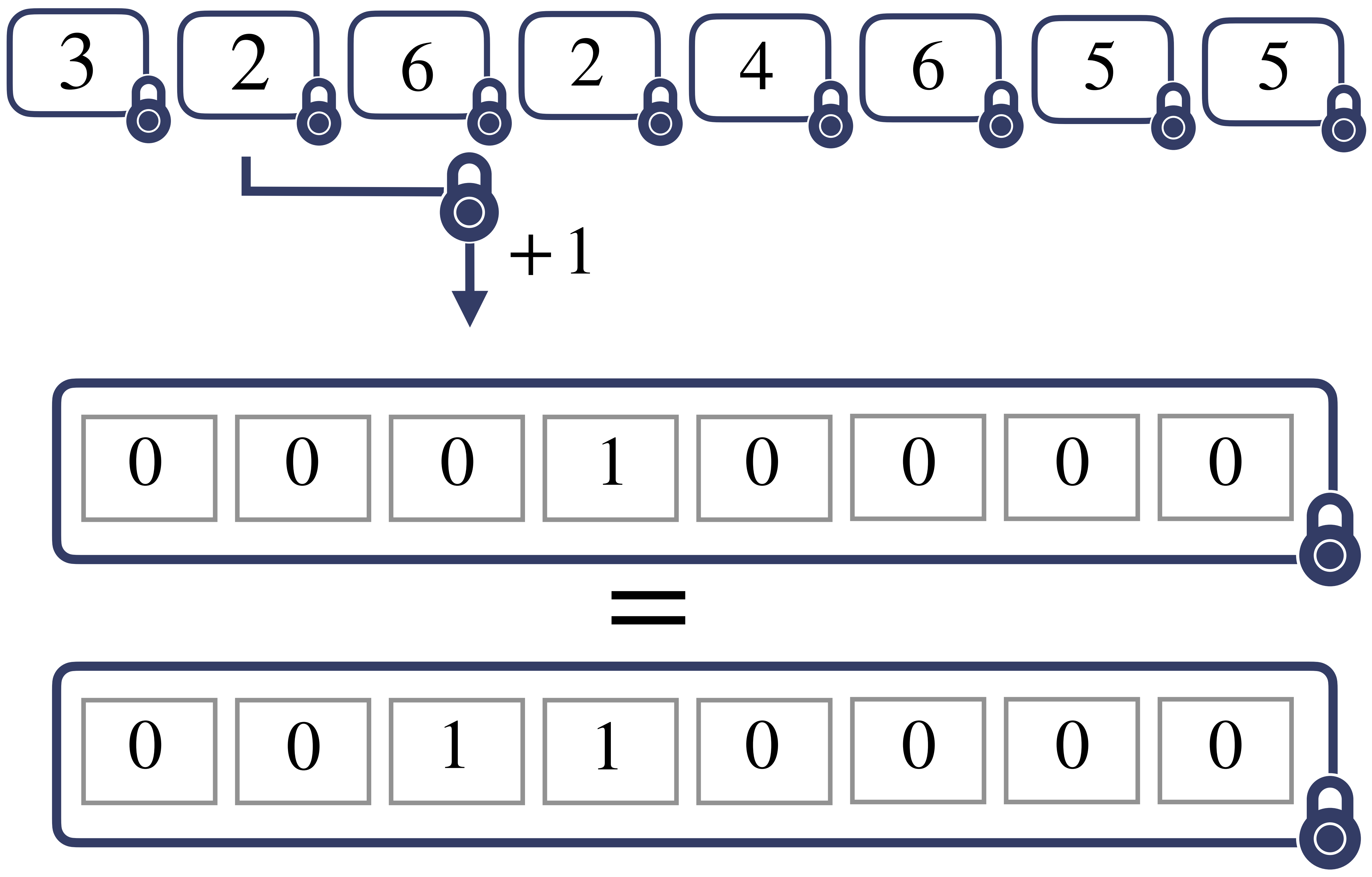

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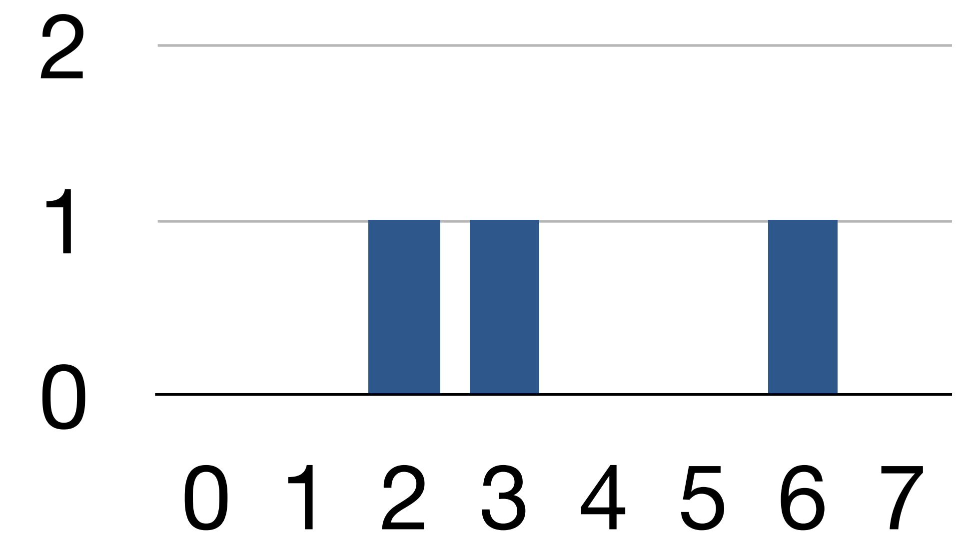

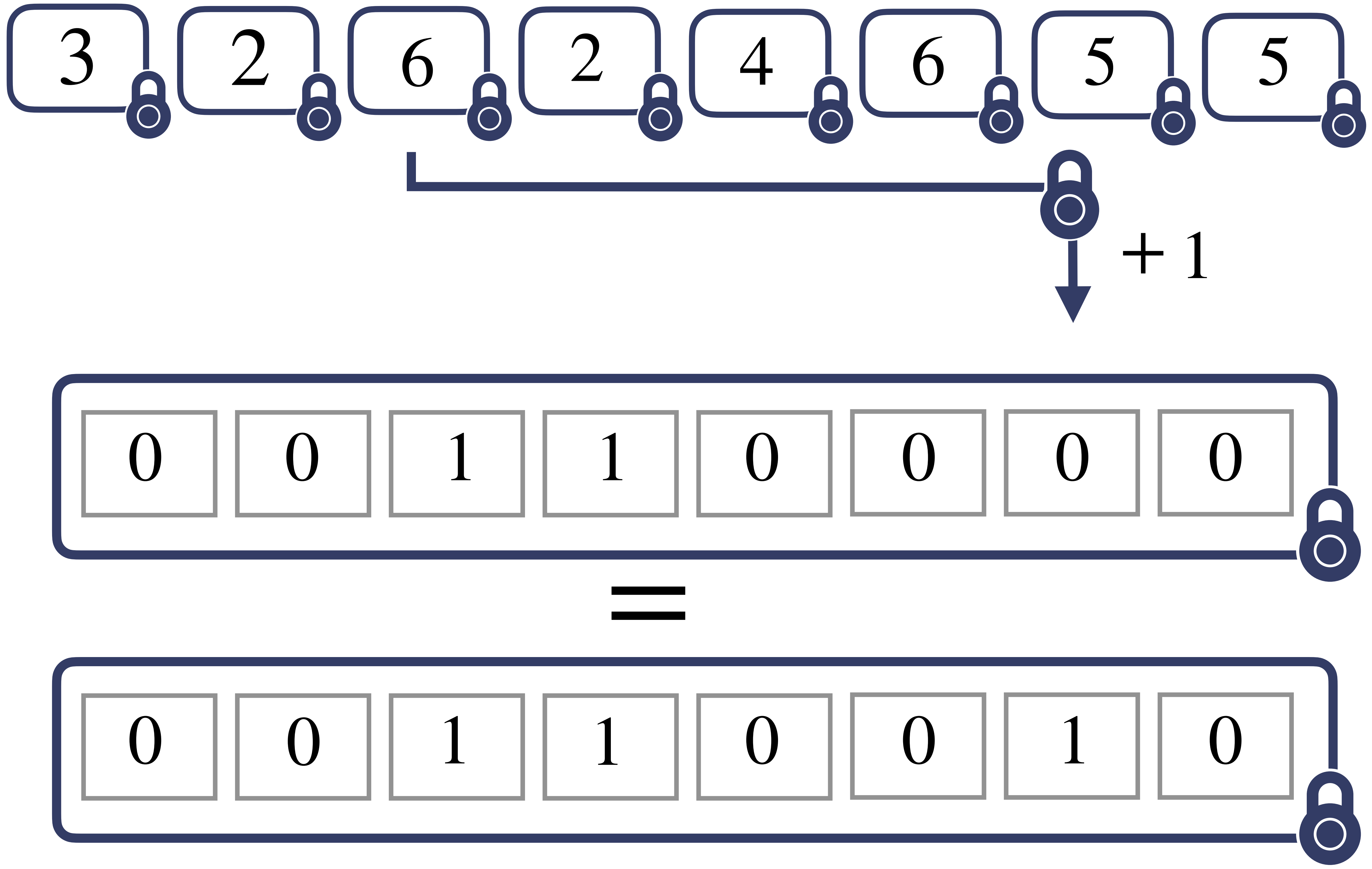

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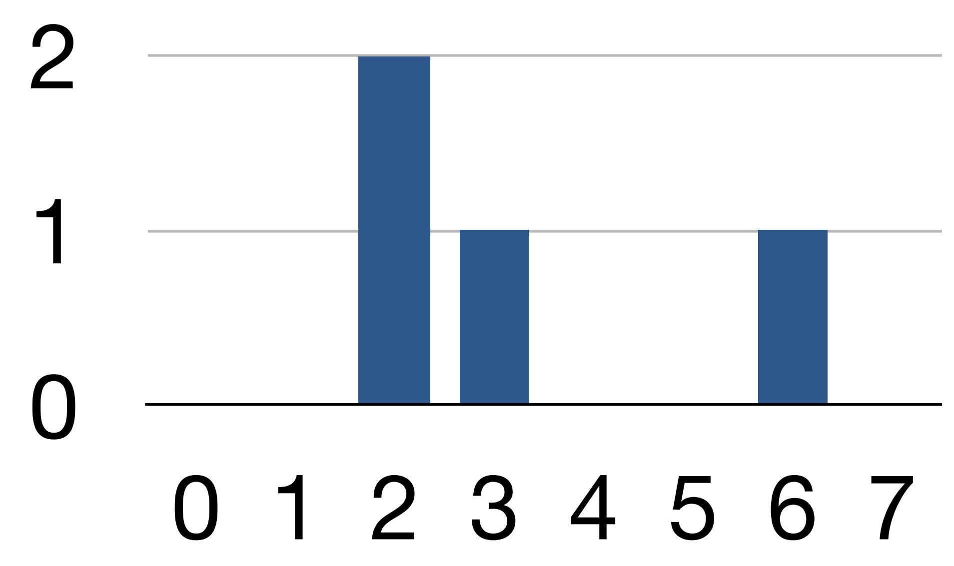

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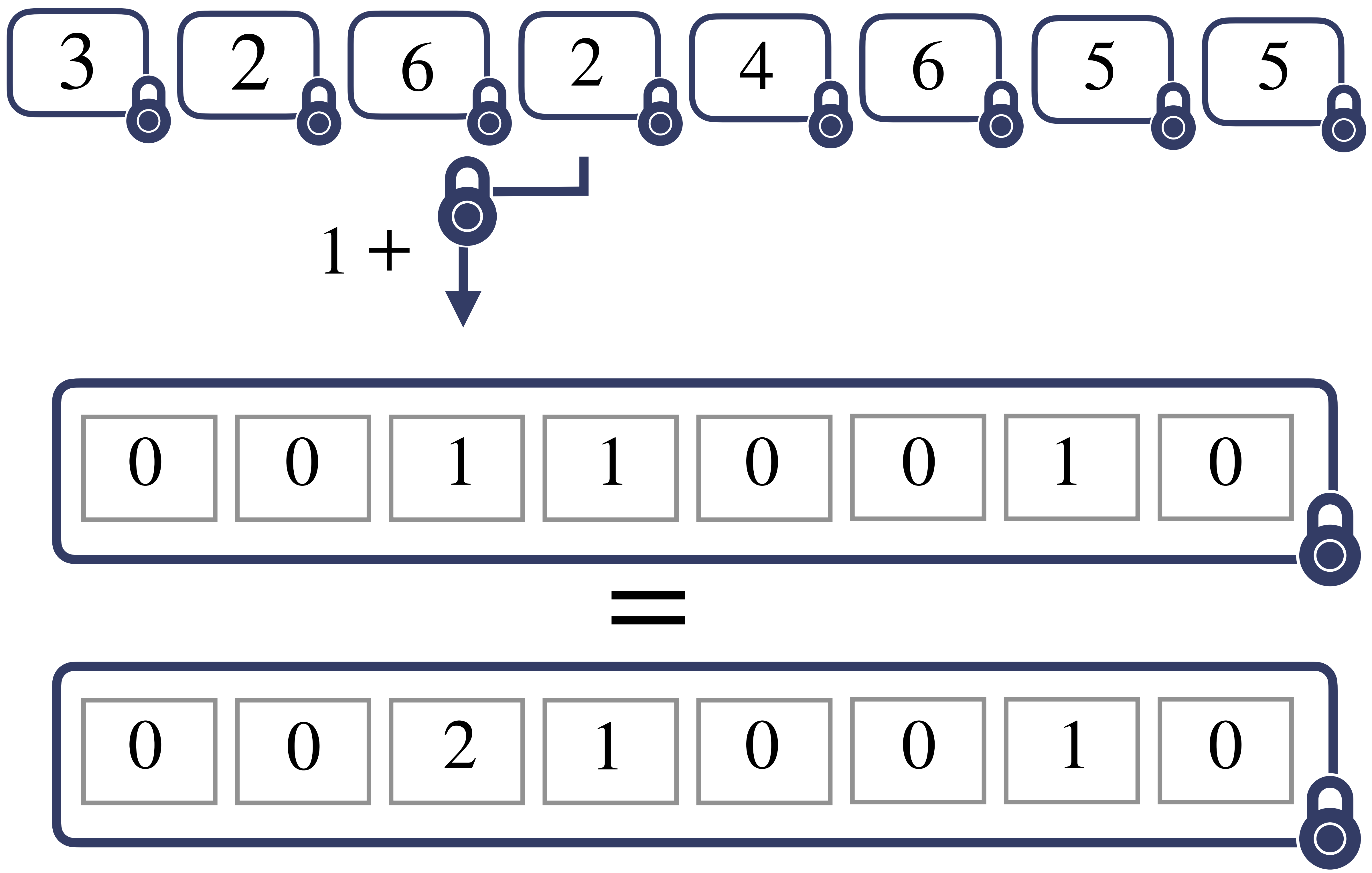

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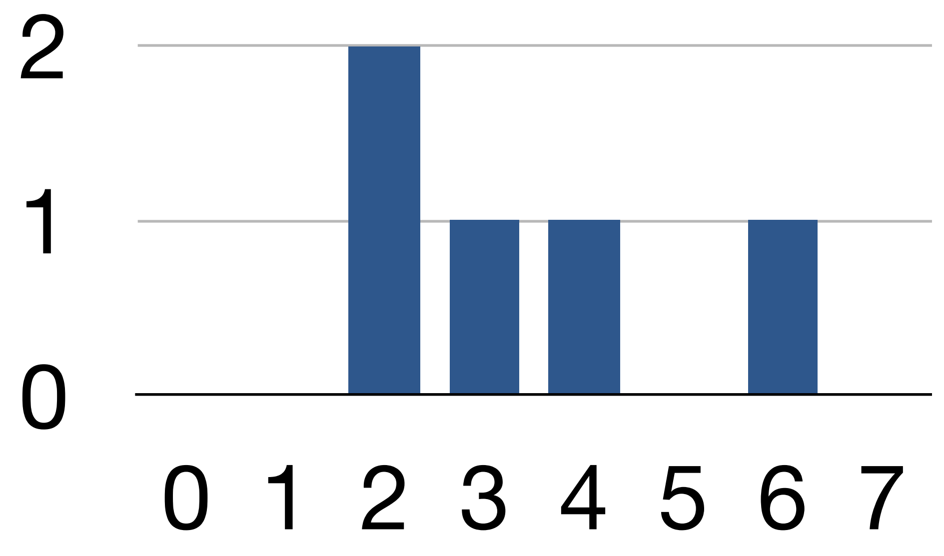

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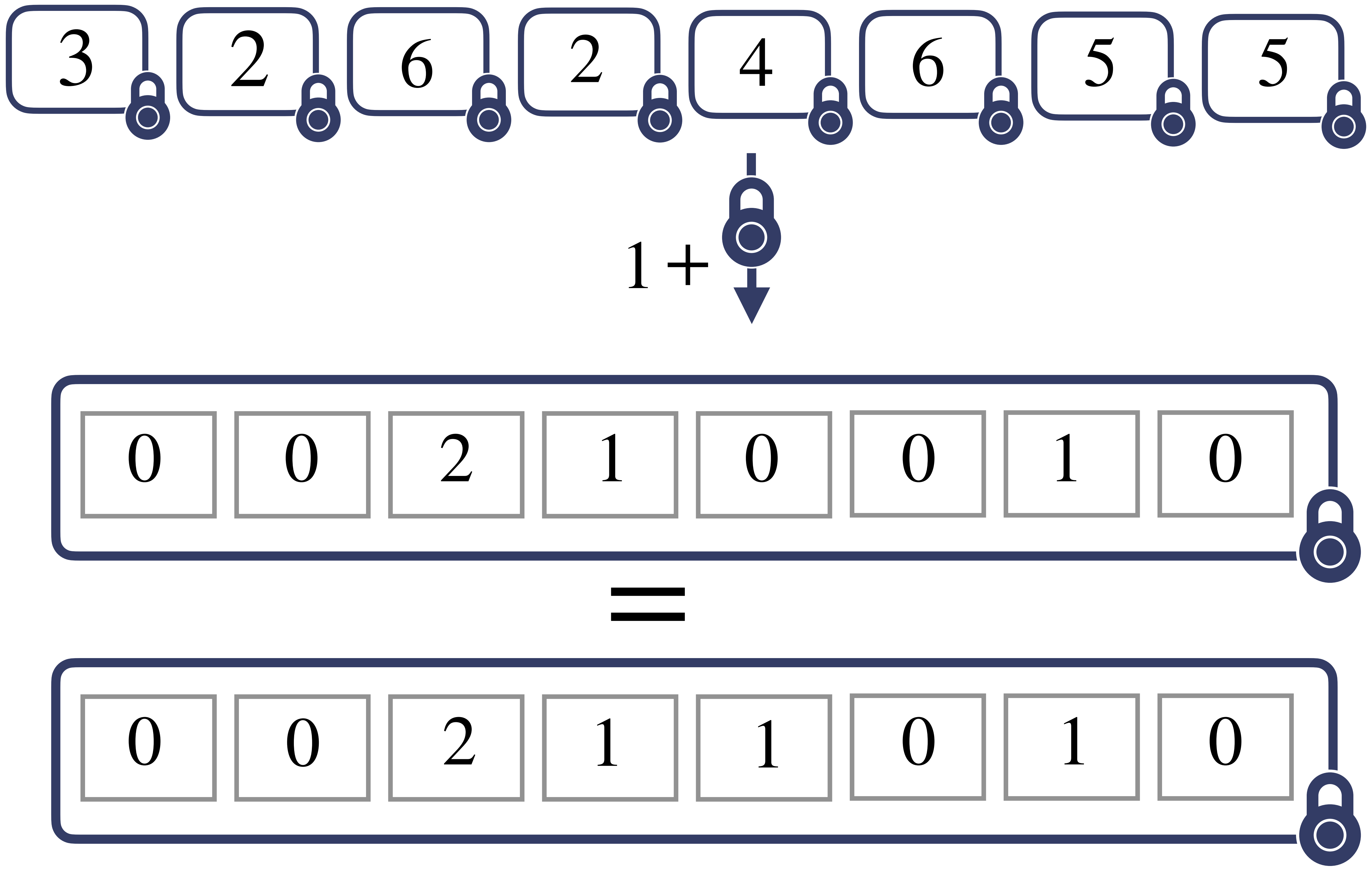

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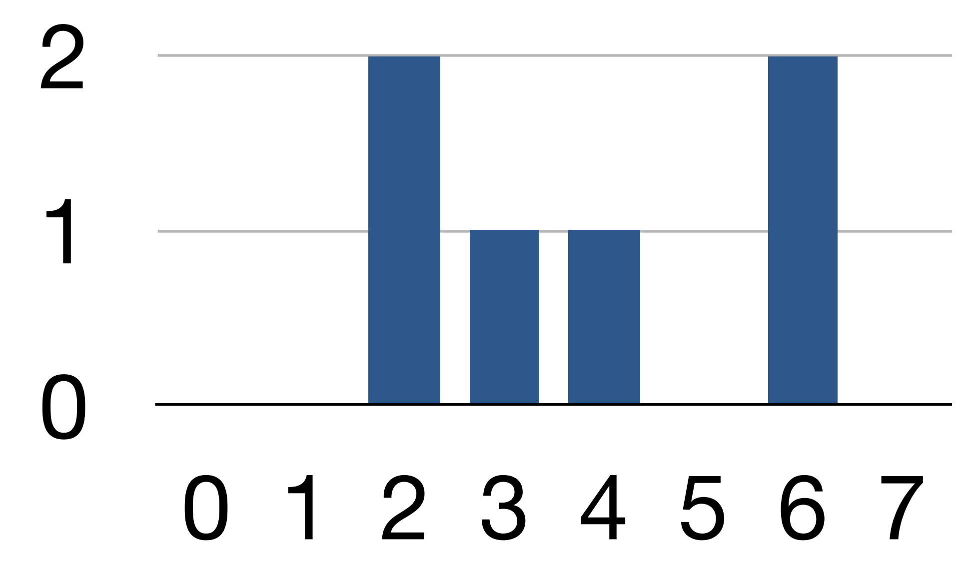

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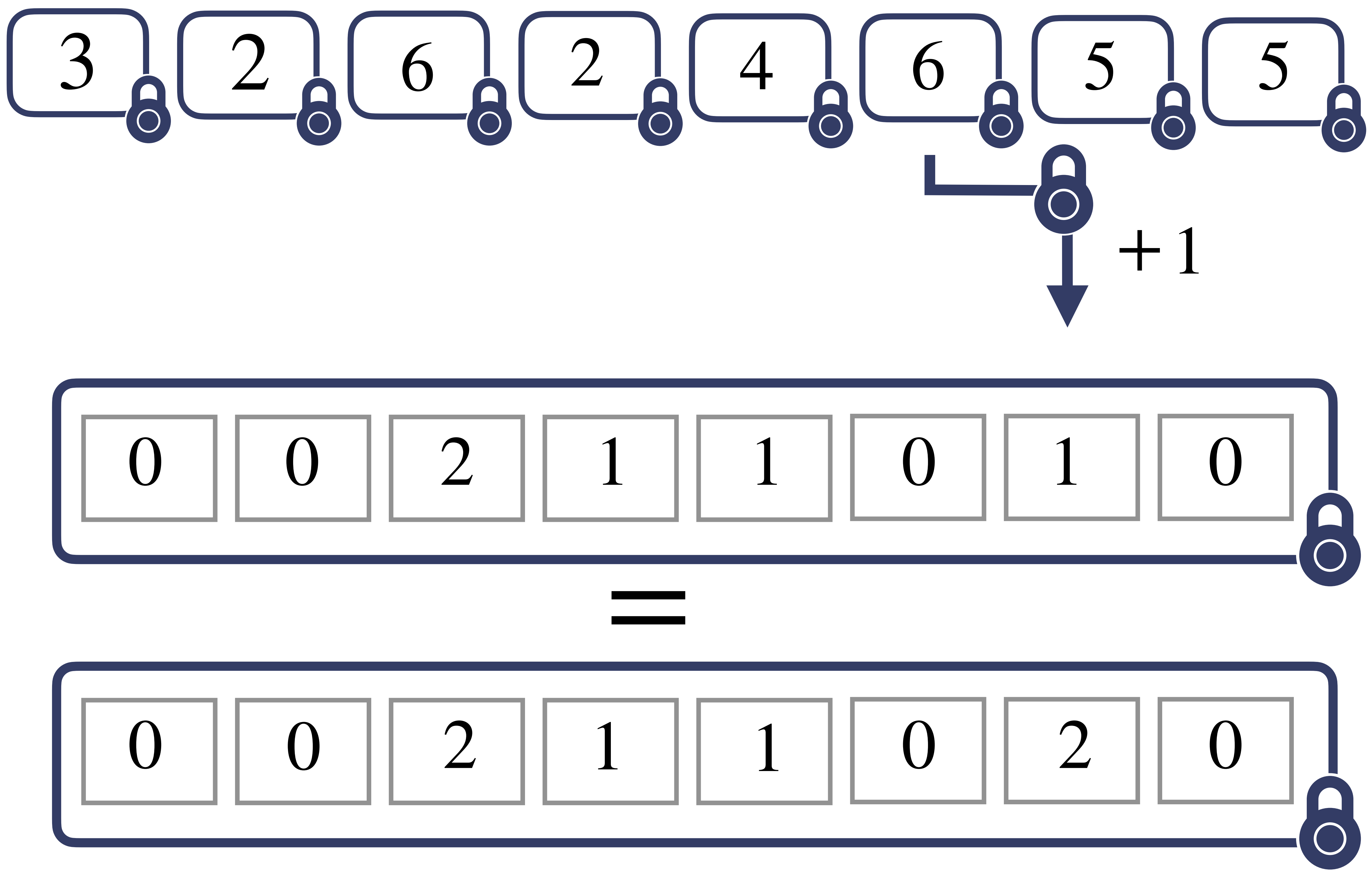

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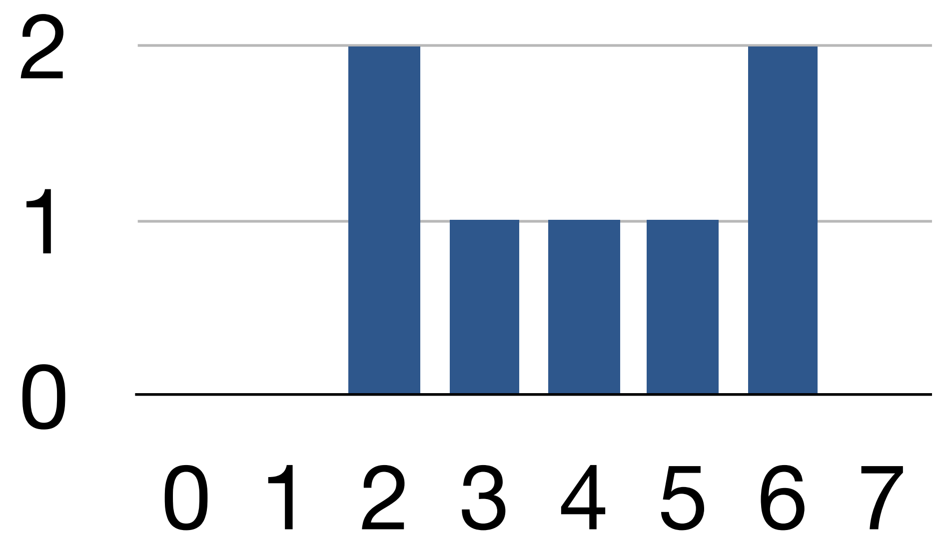

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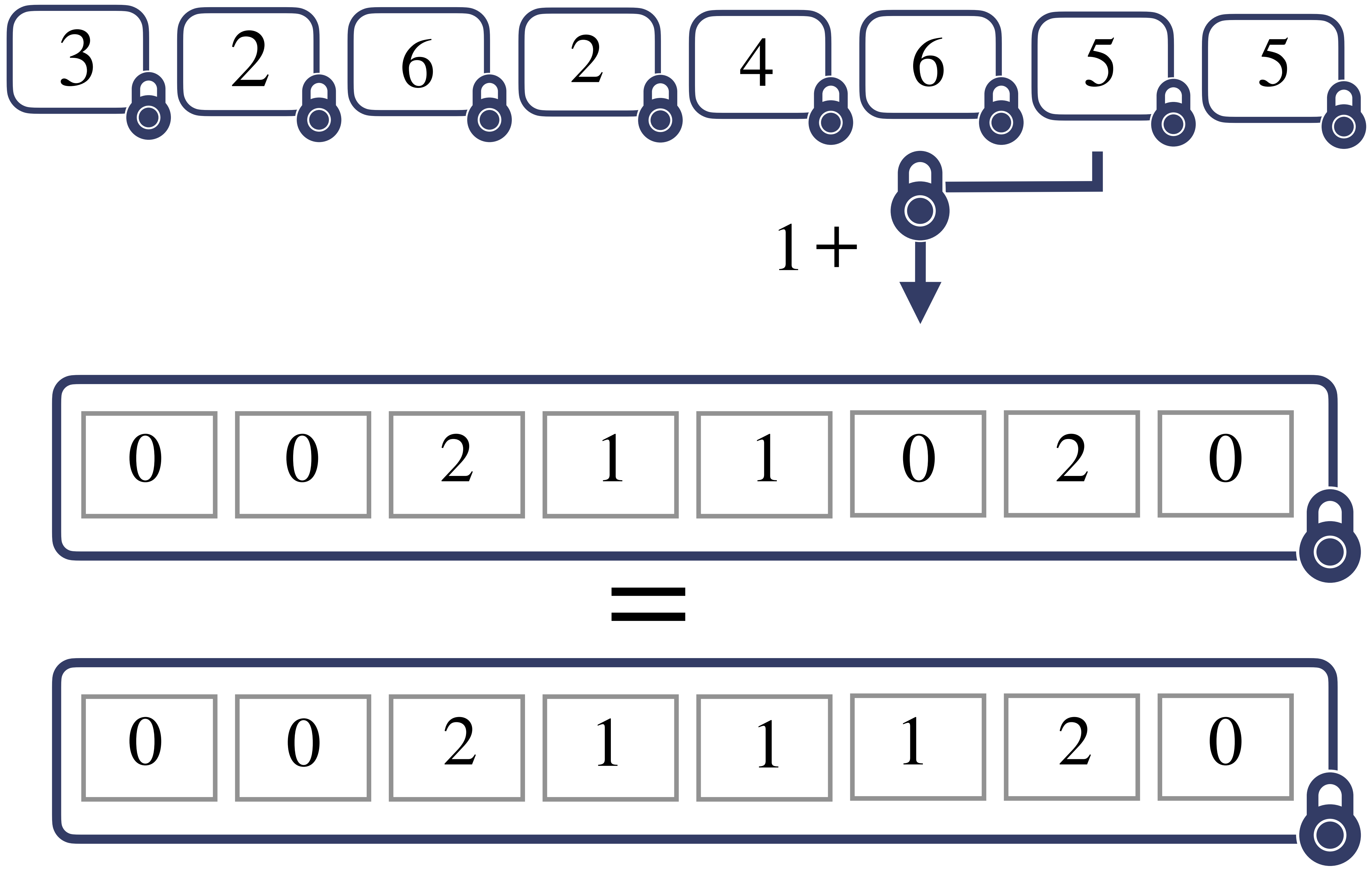

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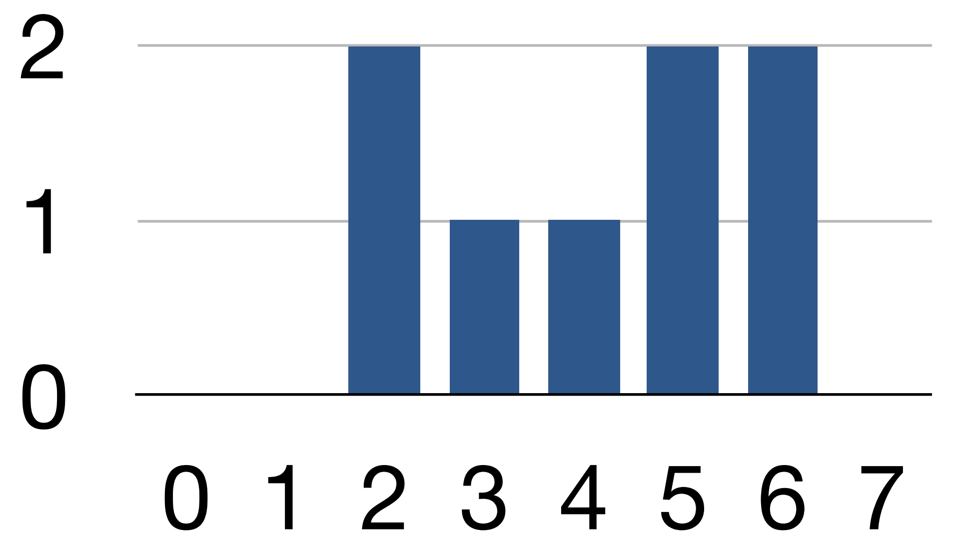

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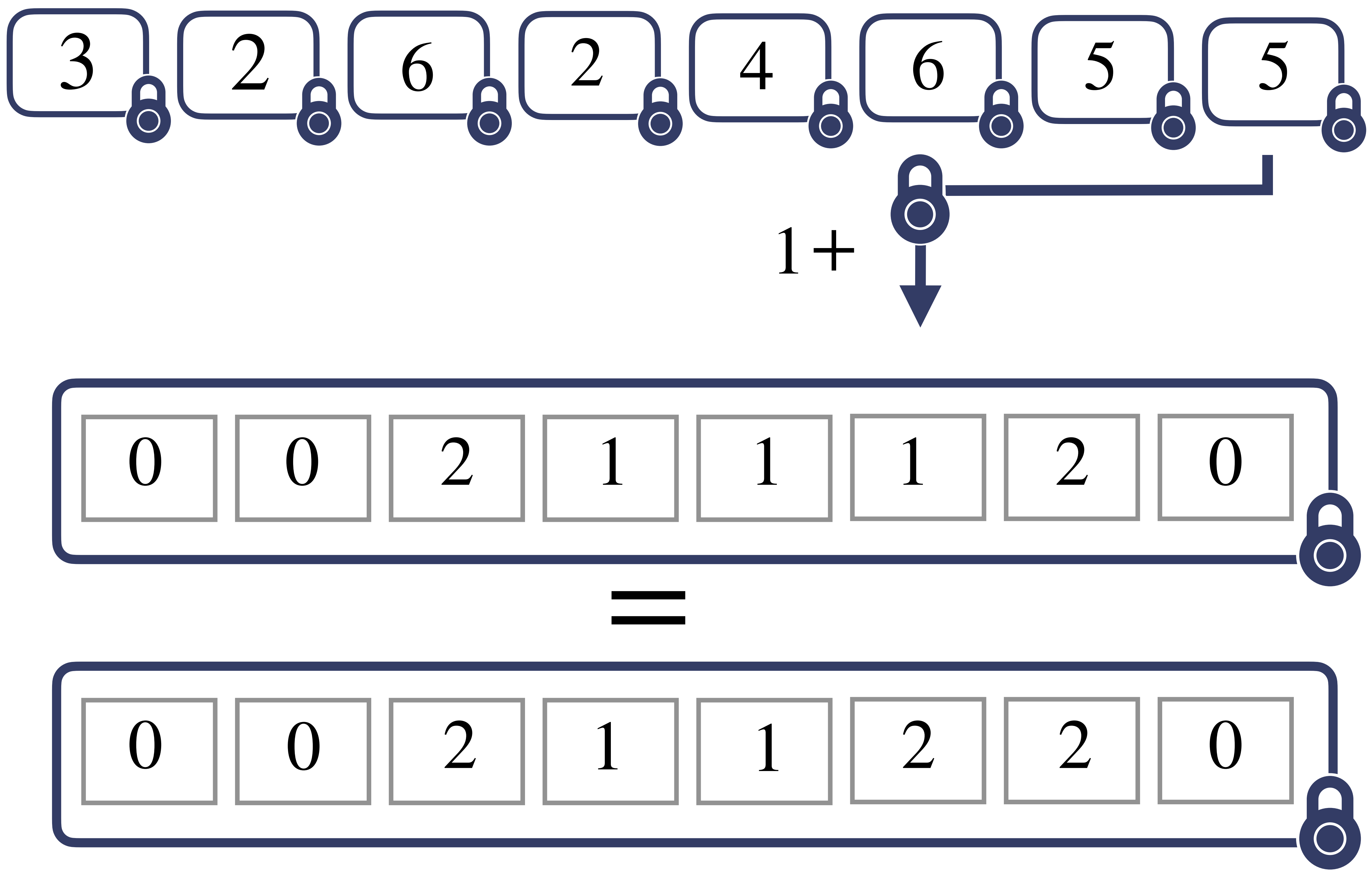

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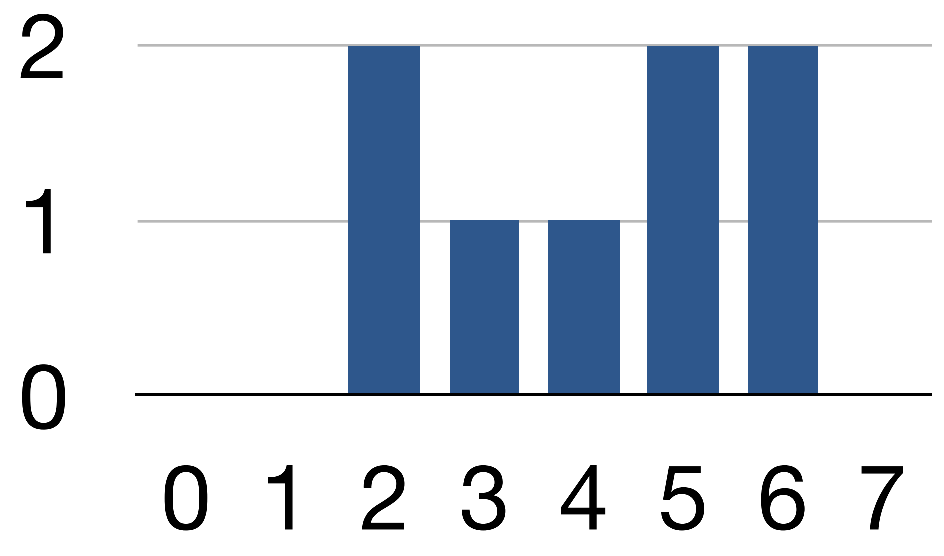
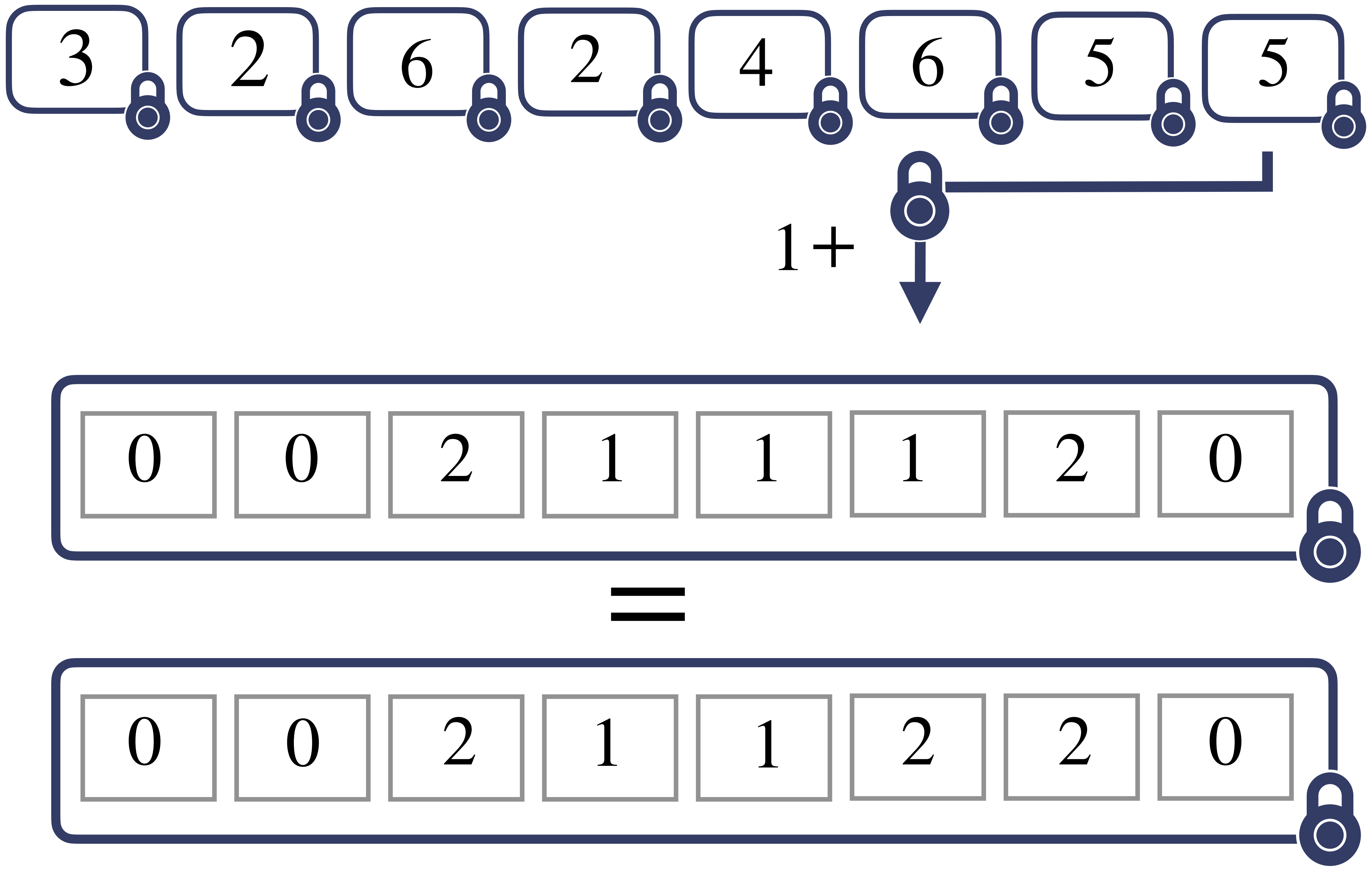


Blind Count

$\mathcal{O}(n)$

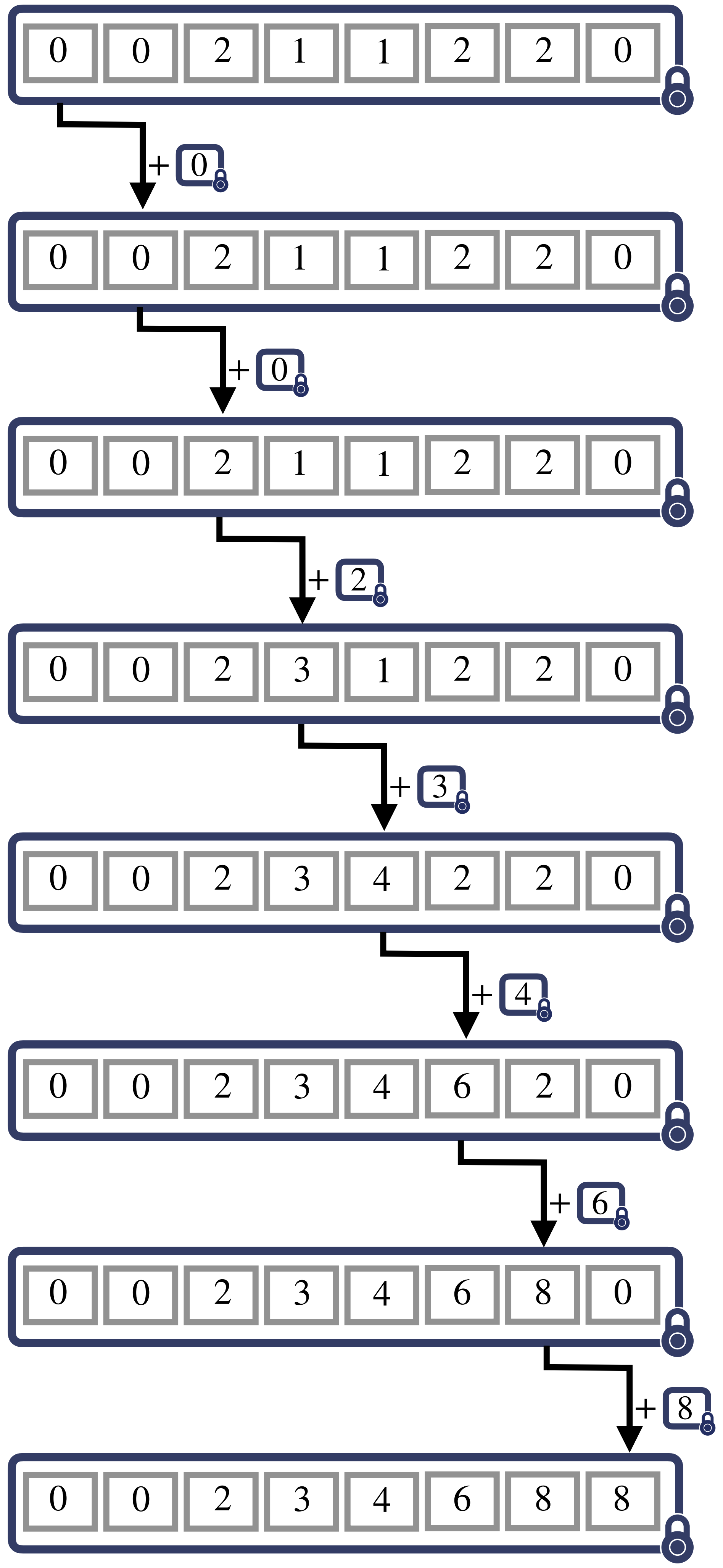
$n \times \text{BR}$

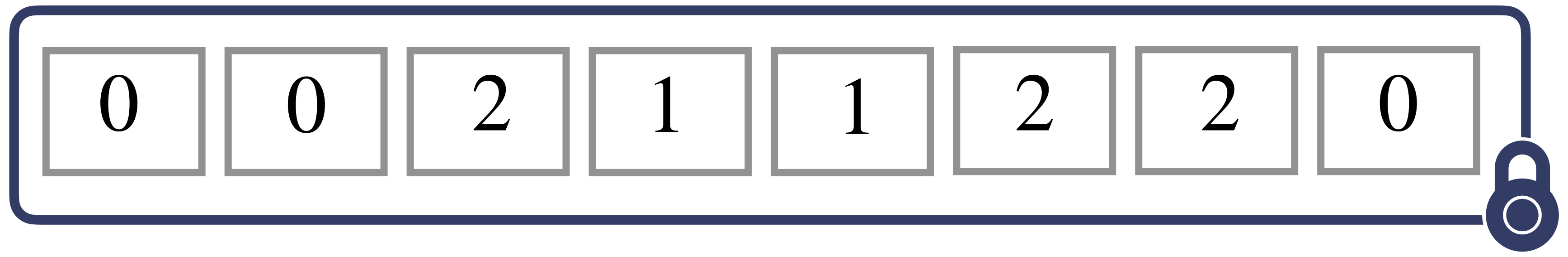


Counts

Blind Counting Sort (BCS)

Build a Prefix Sum





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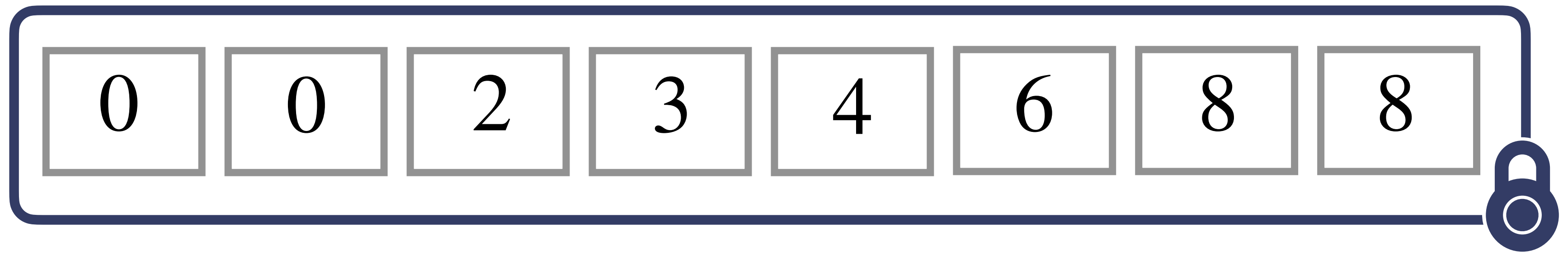

Prefix Sum

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Build the sorted array

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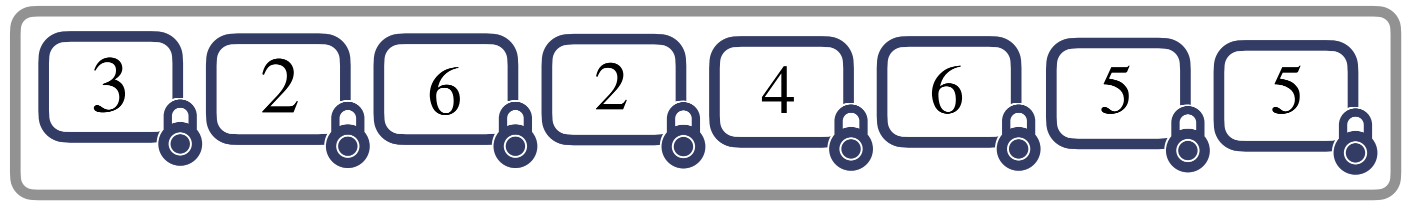

Unsorted array

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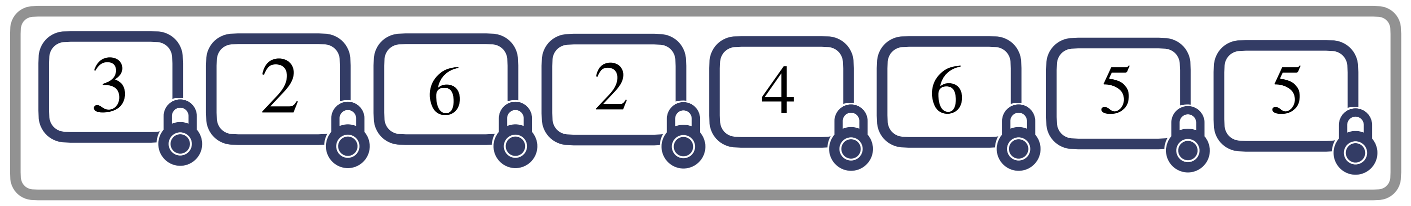

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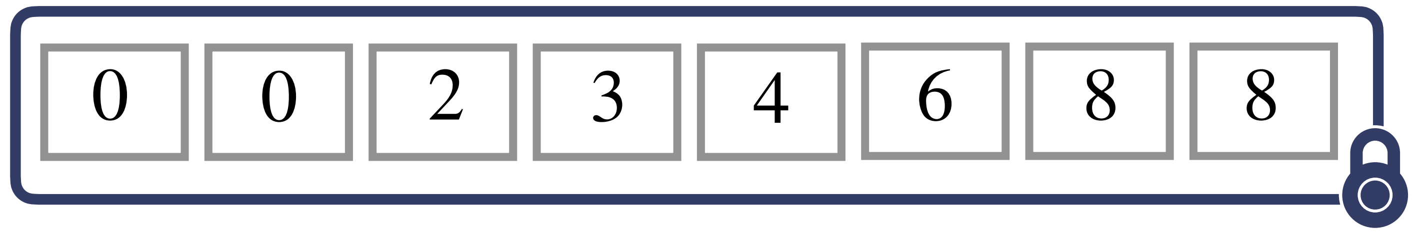

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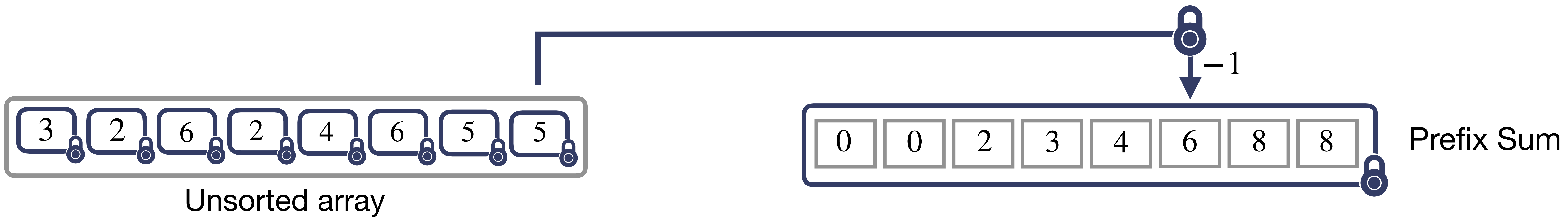

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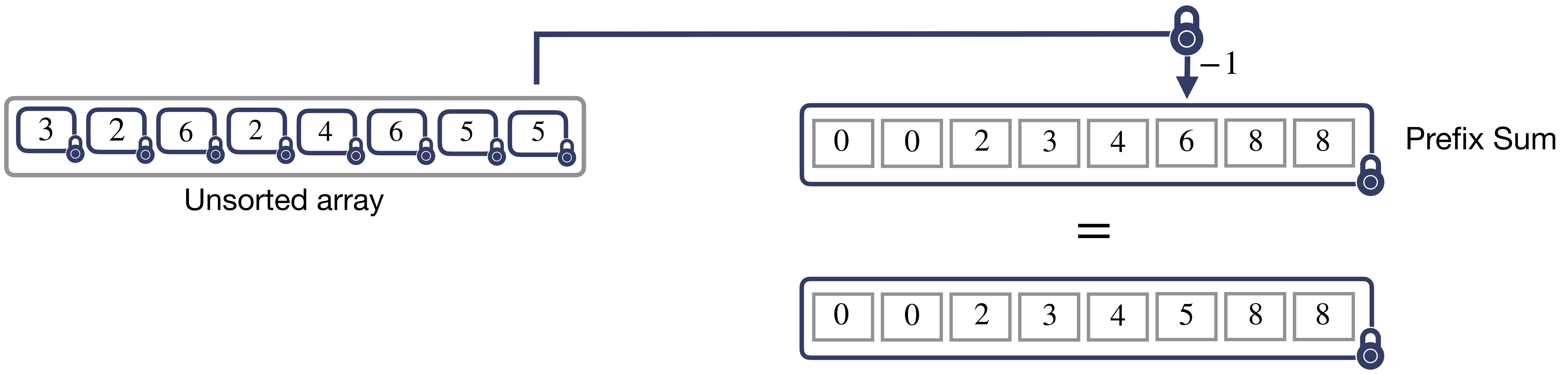

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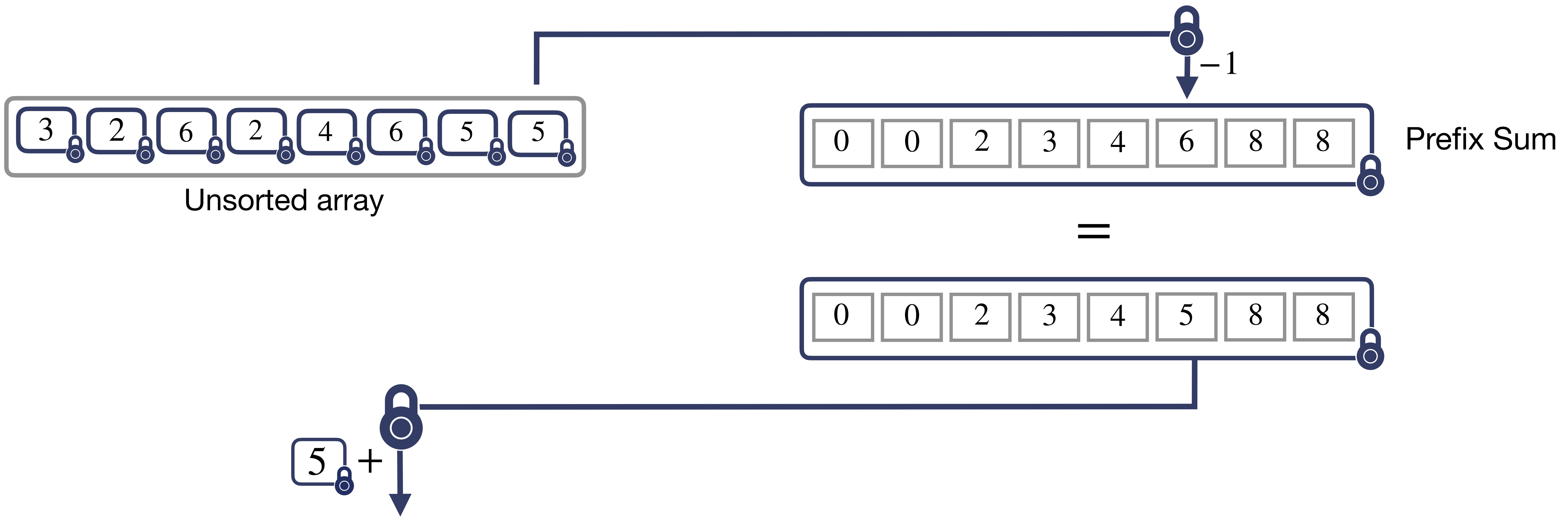

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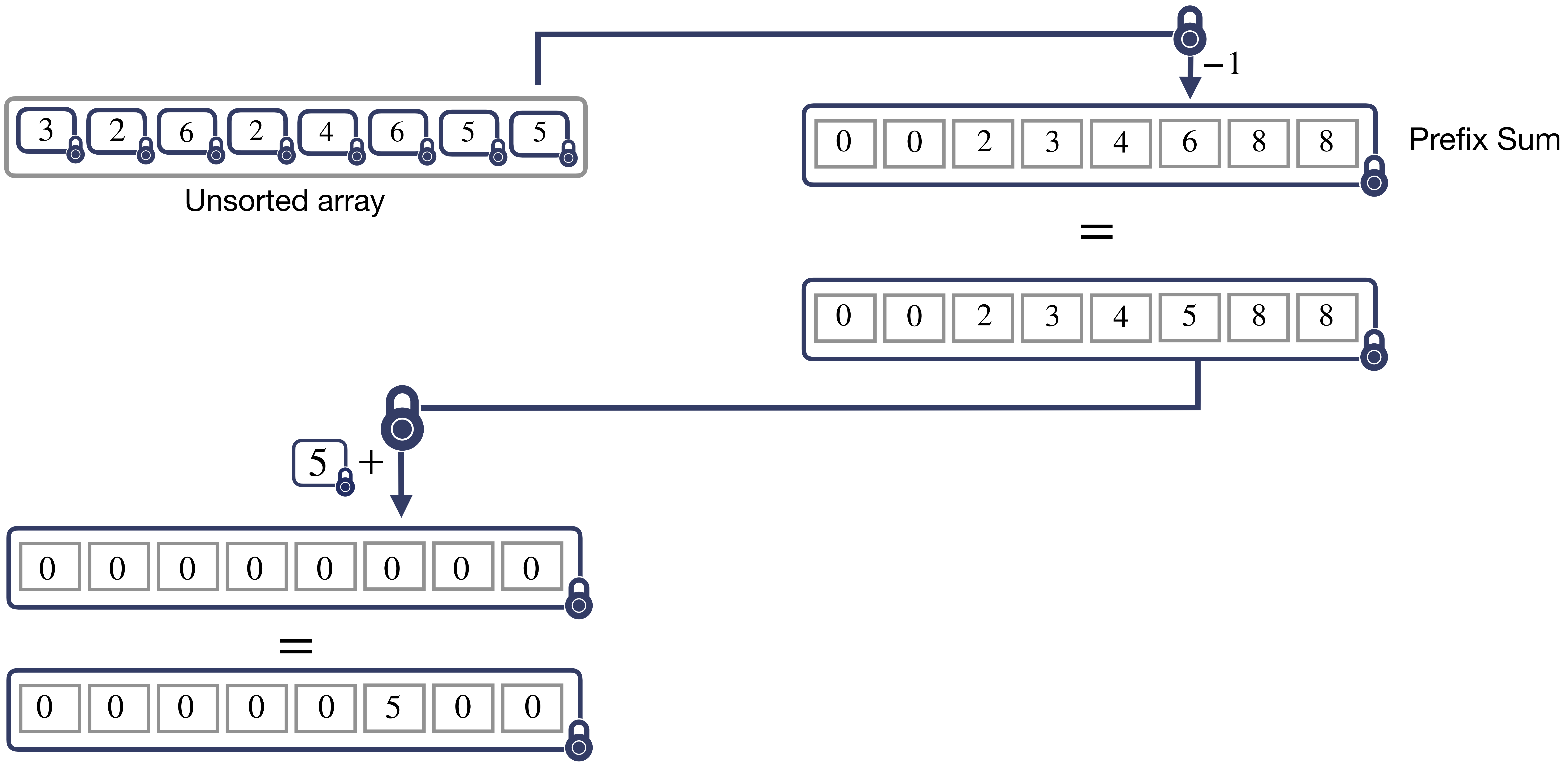

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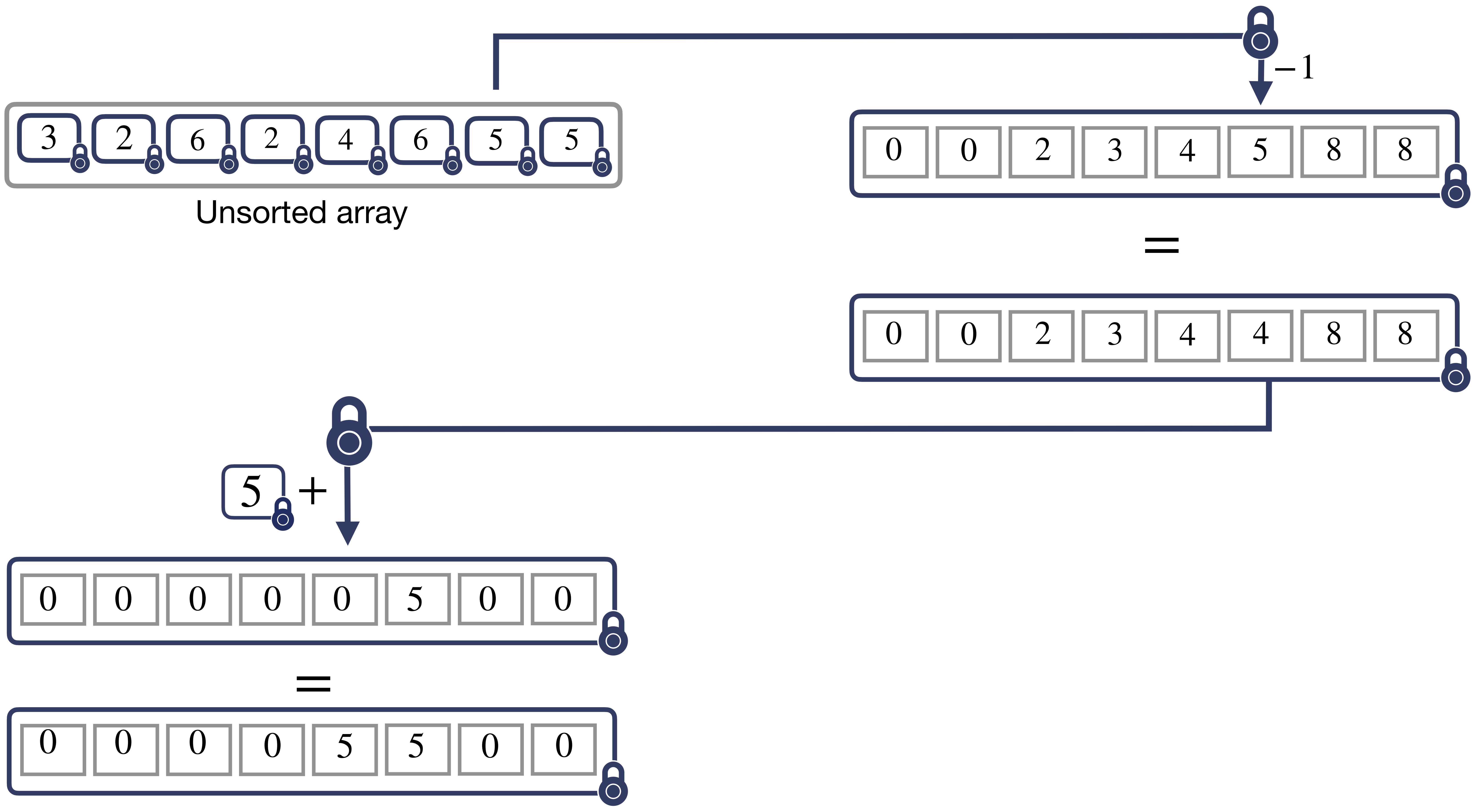

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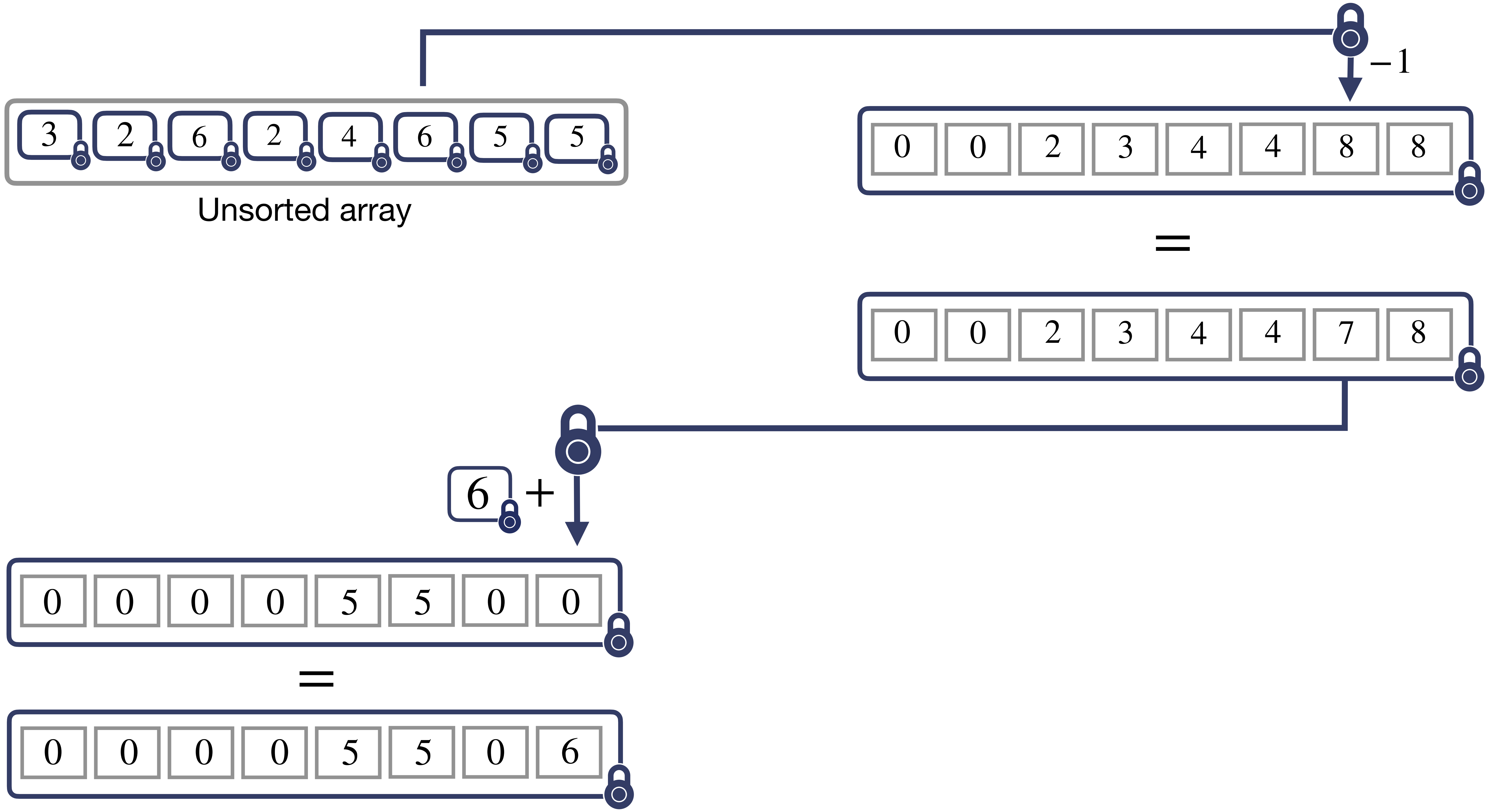

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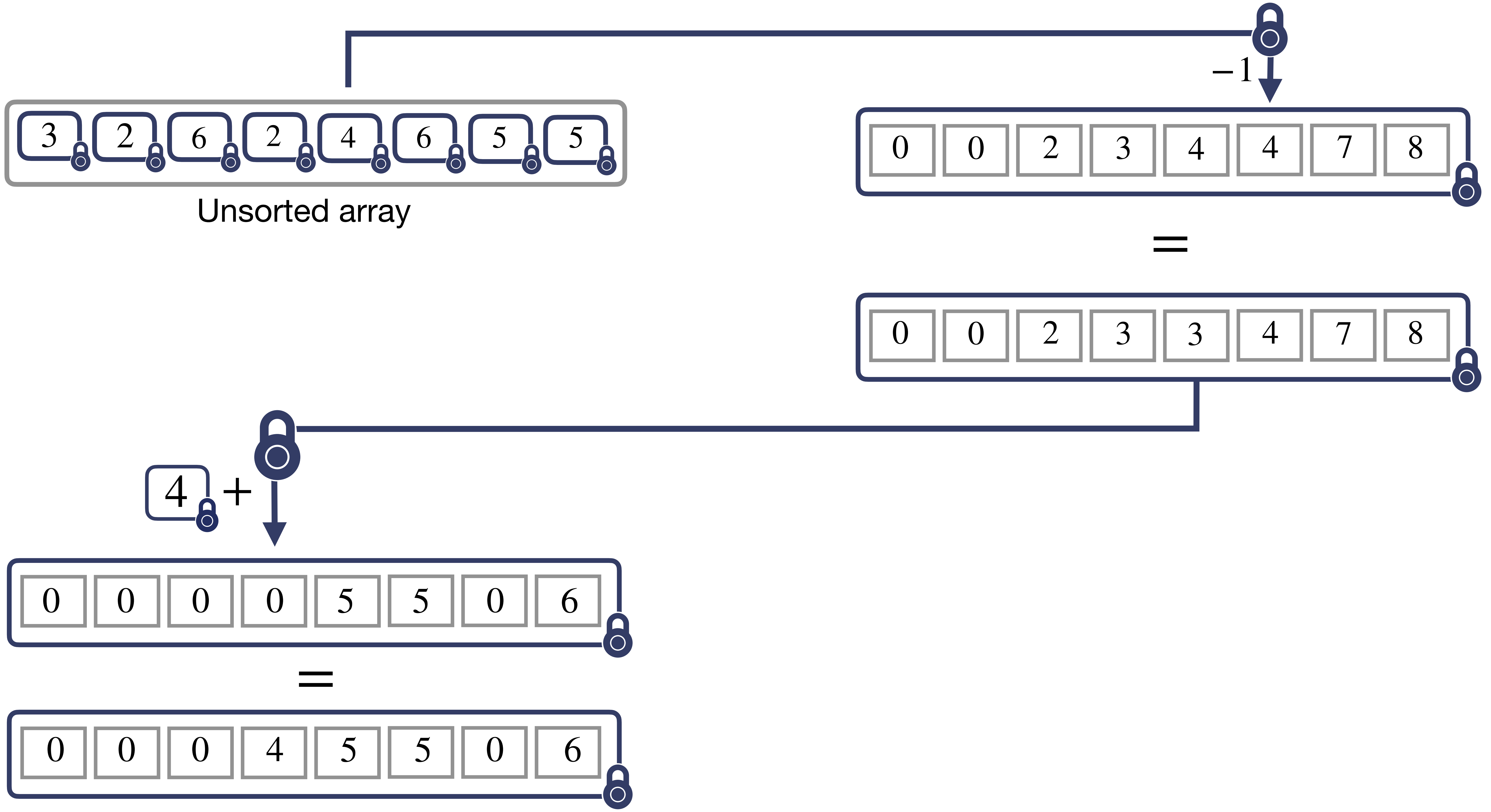

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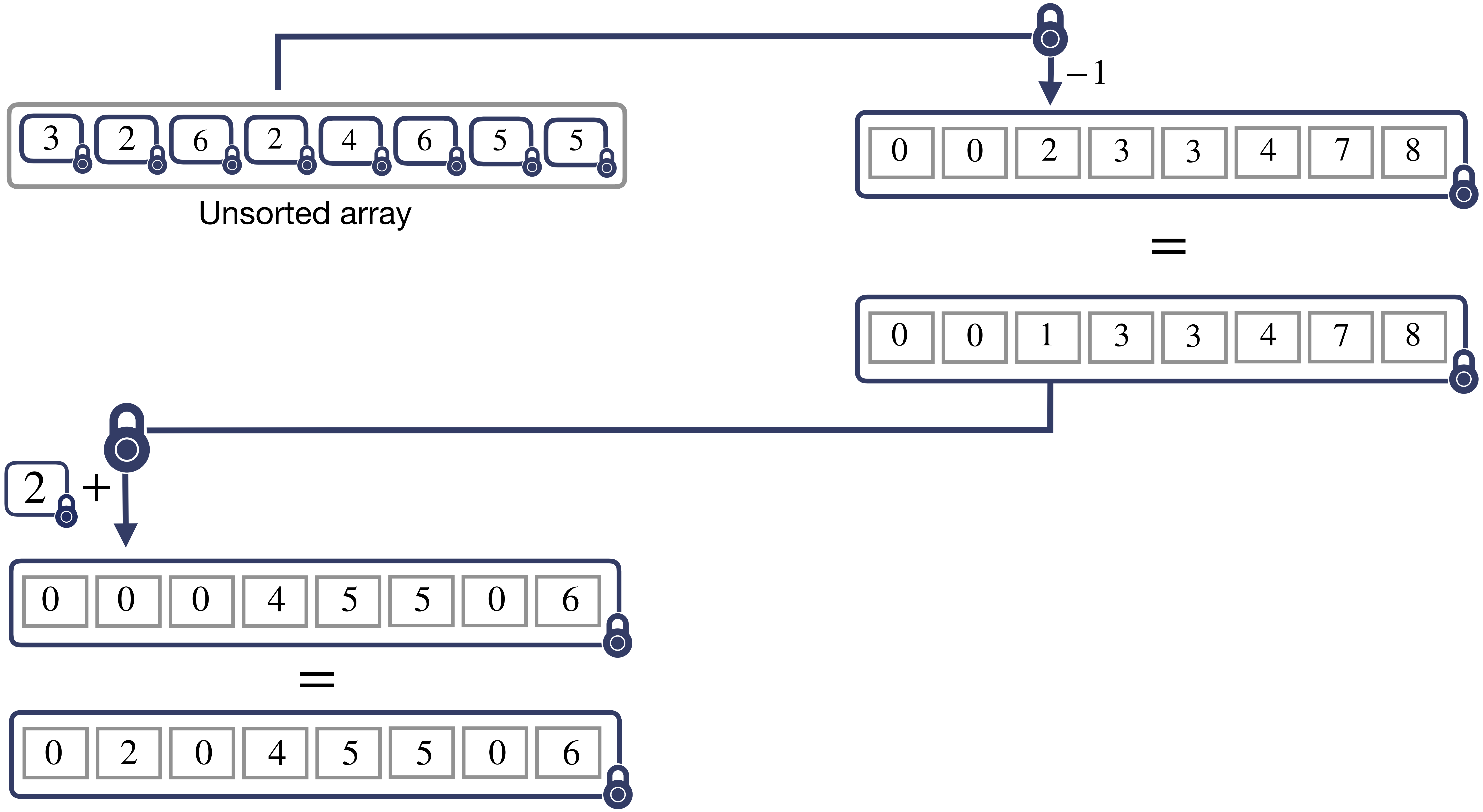

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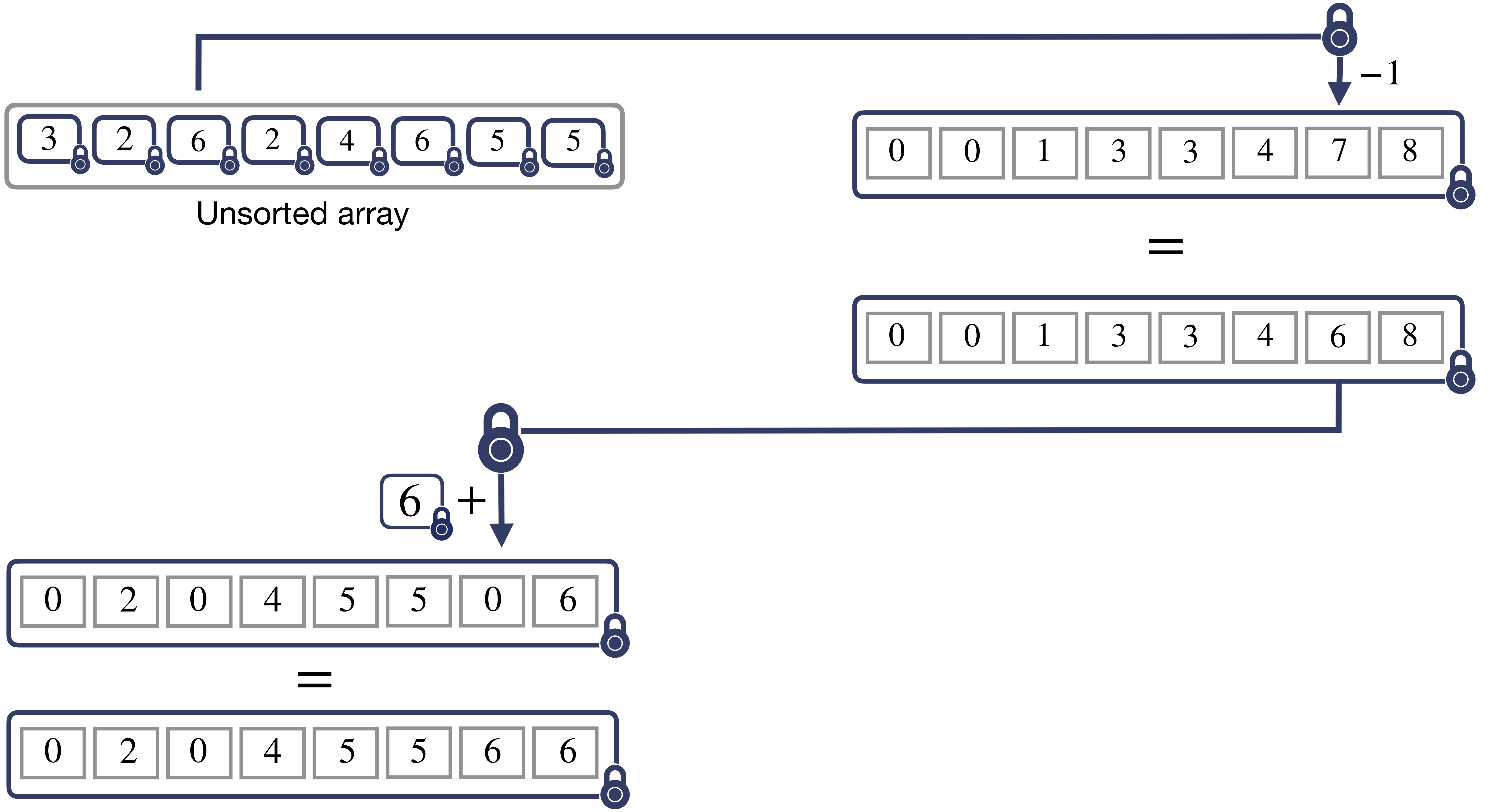

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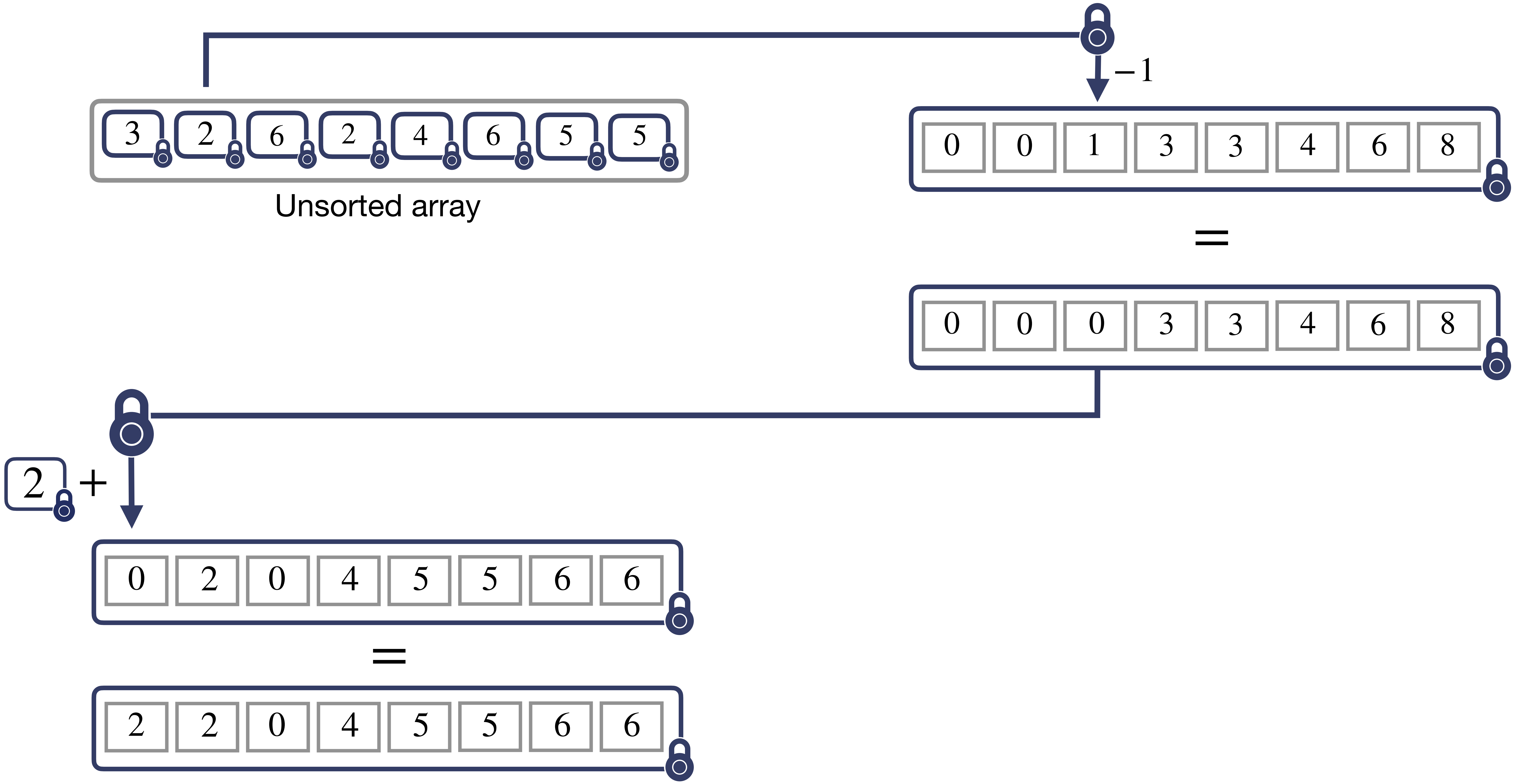

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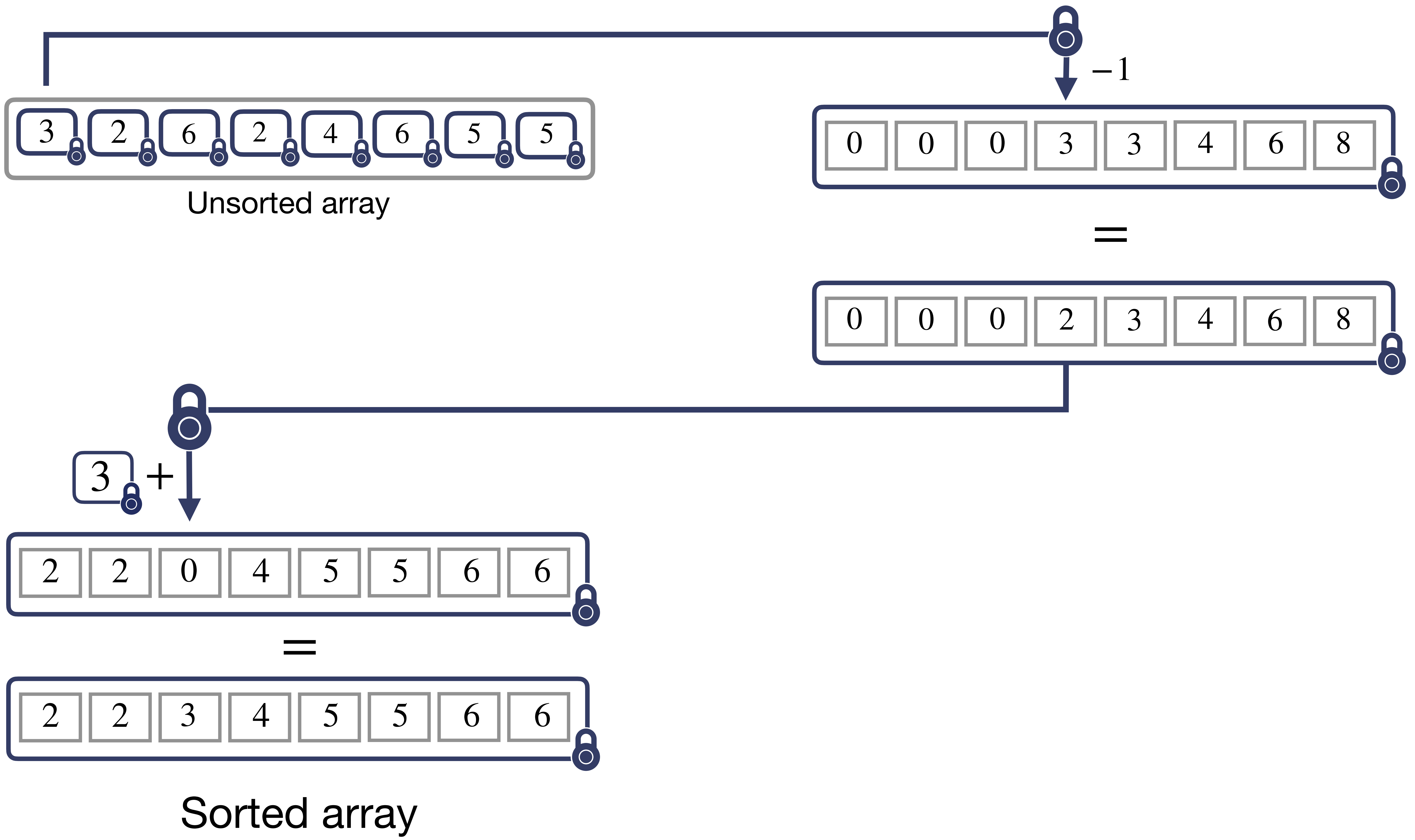
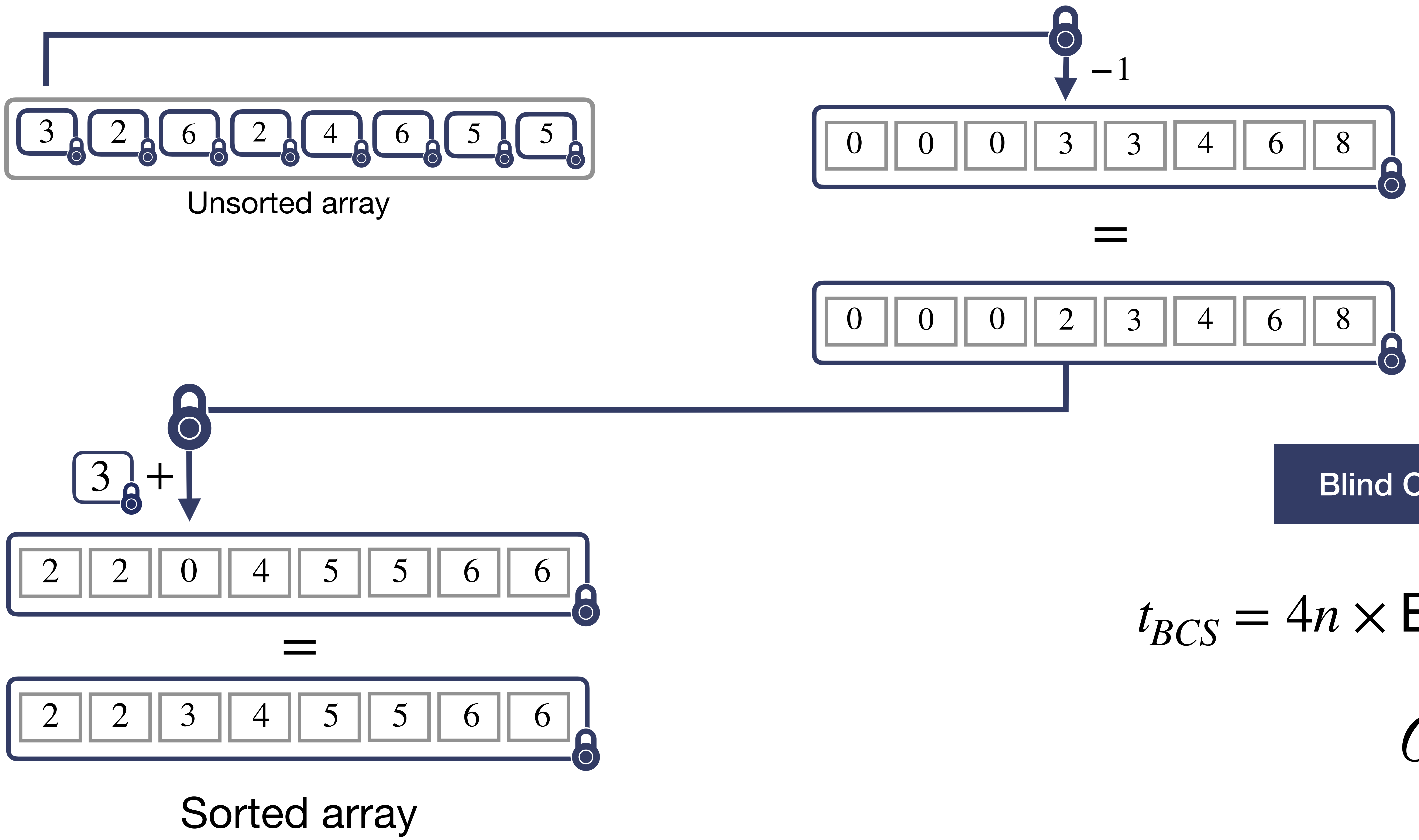


Blind Counting Sort (BCS)

Build the sorted array



Blind Counting Sort

$$t_{BCS} = 4n \times \mathrm{BR} + 2n \times \mathrm{PFKS}$$

$$\mathcal{O}(n)$$

Blind Counting Sort (BCS)

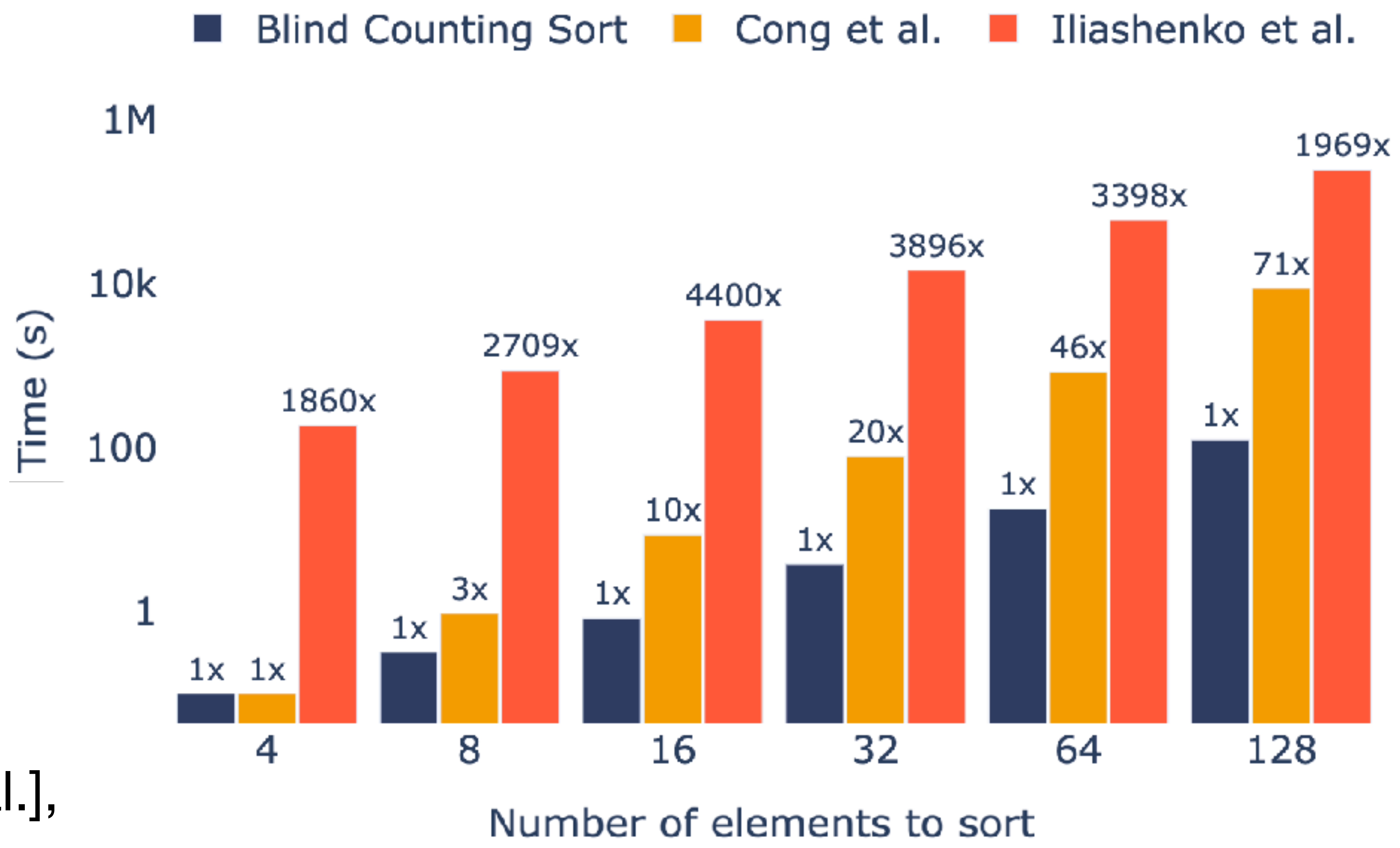
Comparison with the SOA

Iliashenko-Zucca. :

- Direct Sort ($\mathcal{O}(n^2)$)
- BGV scheme (SIMD compatible)
- Faster comparison

Cong et al. :

- Batcher Sort ($\mathcal{O}(nlog(n))$)
- TFHE scheme
- Build on the comparison method by [Zuber et al.], which needs **an extra bit of precision**.



[Iliashenko-Zucca.] : Faster homomorphic comparison operations for BGV and BFV, PoPETs, 2021(3), 246–264.

[Cong et al.] : Revisiting Oblivious Top-k Selection with Applications to Secure k-NN Classification, SAC 2024, LNCS 15516.

[Zuber et al.] : Efficient and accurate homomorphic comparisons, WHAC 2022, pp. 35–46.

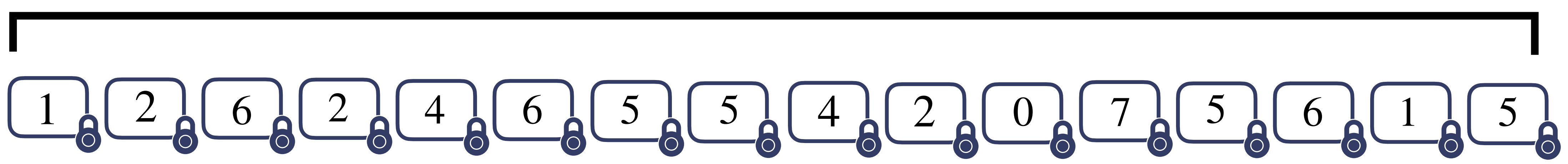
Blind Top-k

Blind Top-k

BCS used in a tournament

Blind Top-k

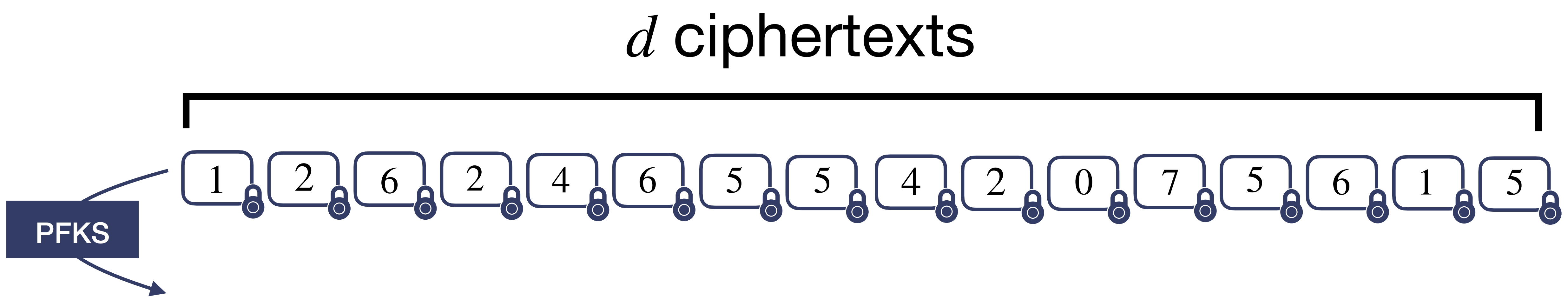
BCS used in a tournament



Blind Top-k

BCS used in a tournament

$k = 3$



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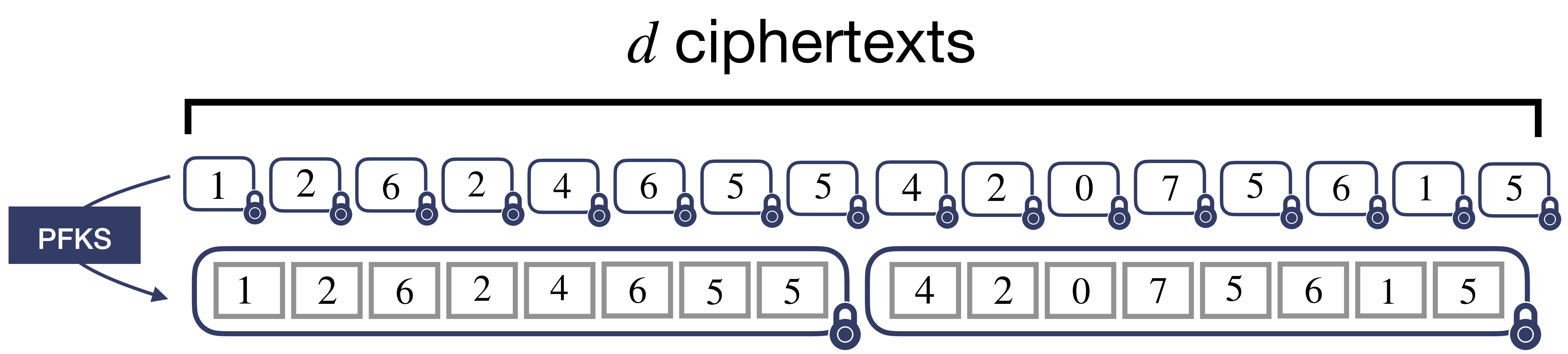

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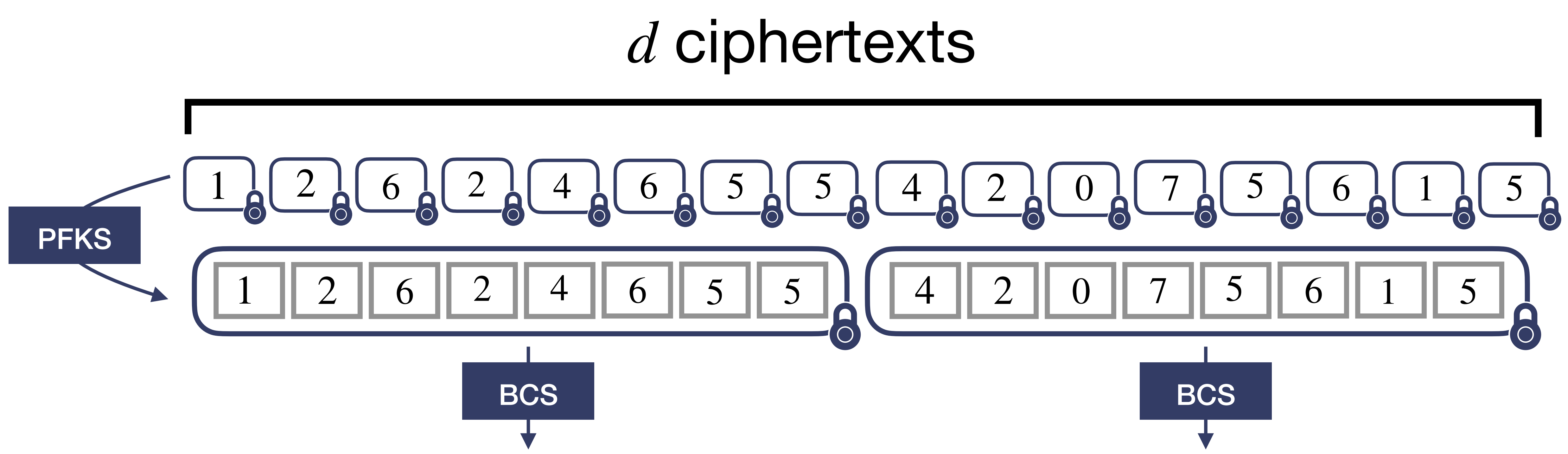

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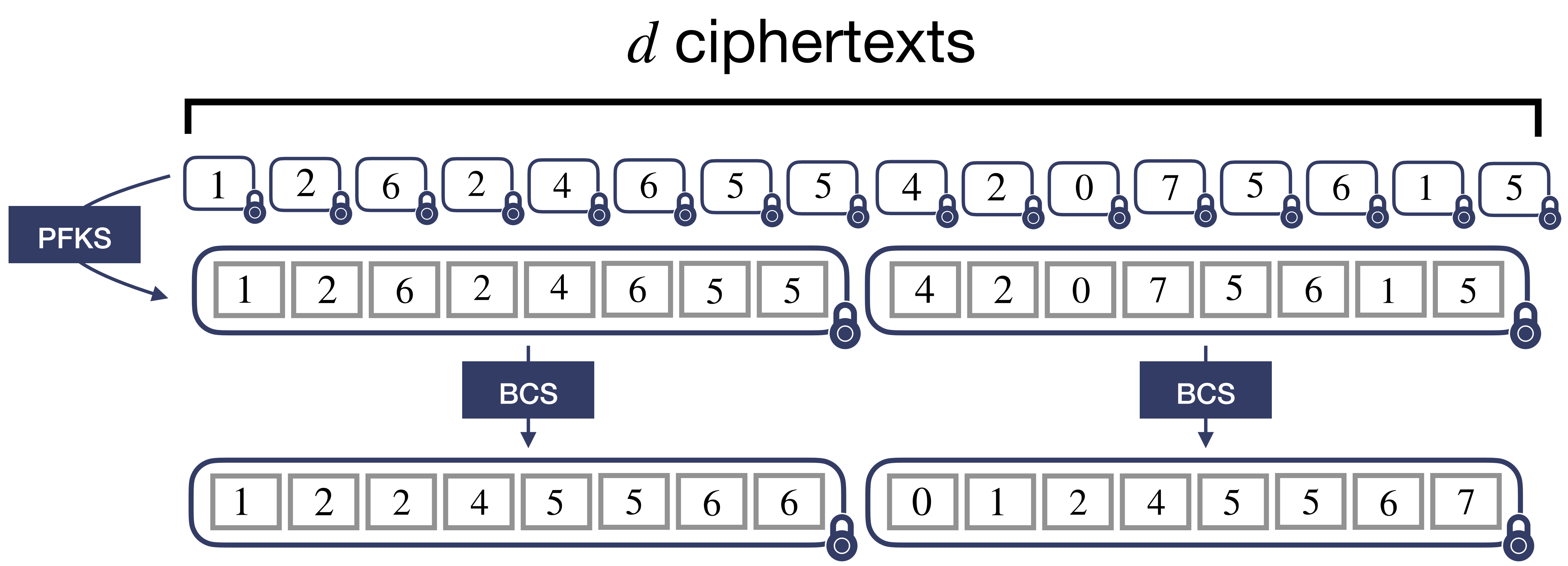

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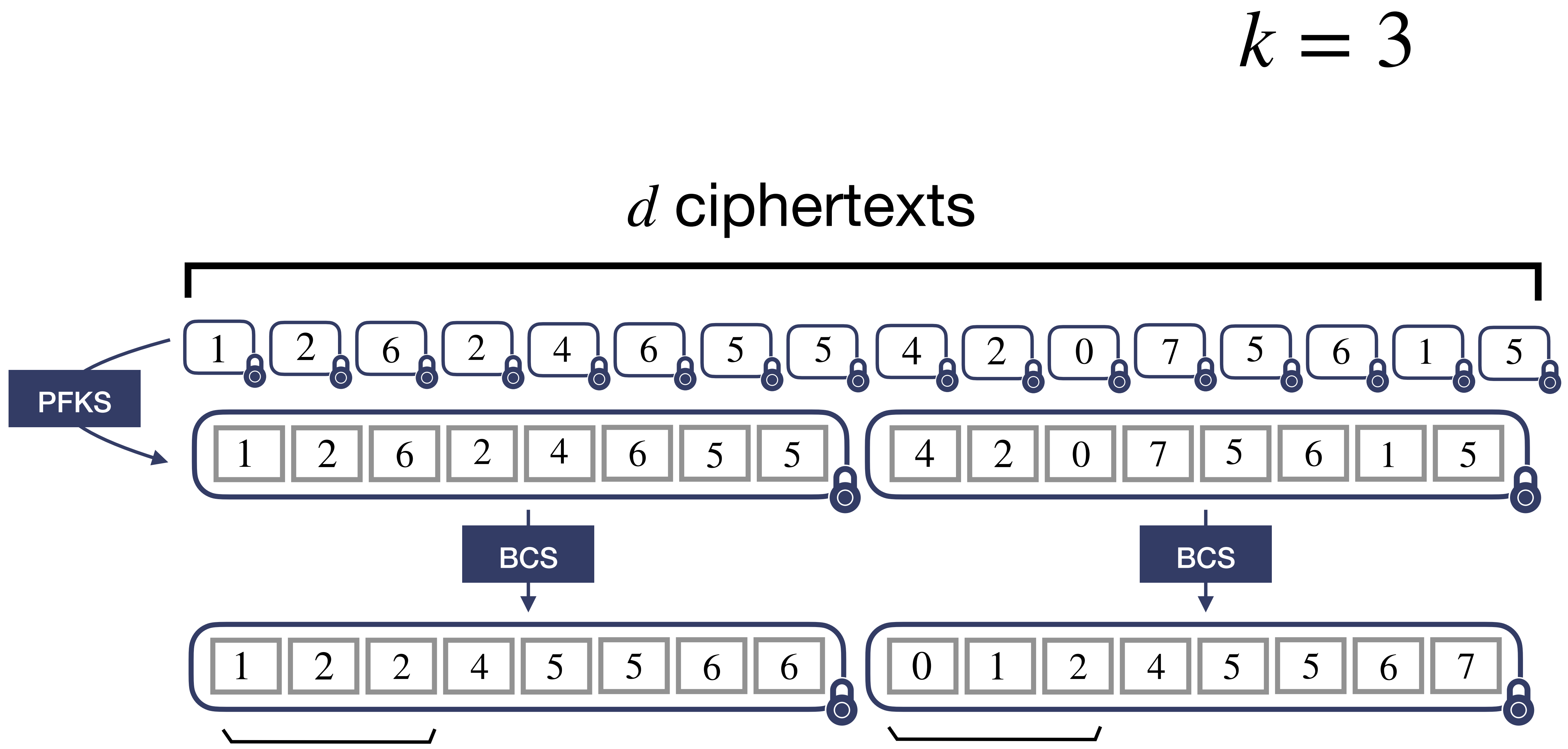

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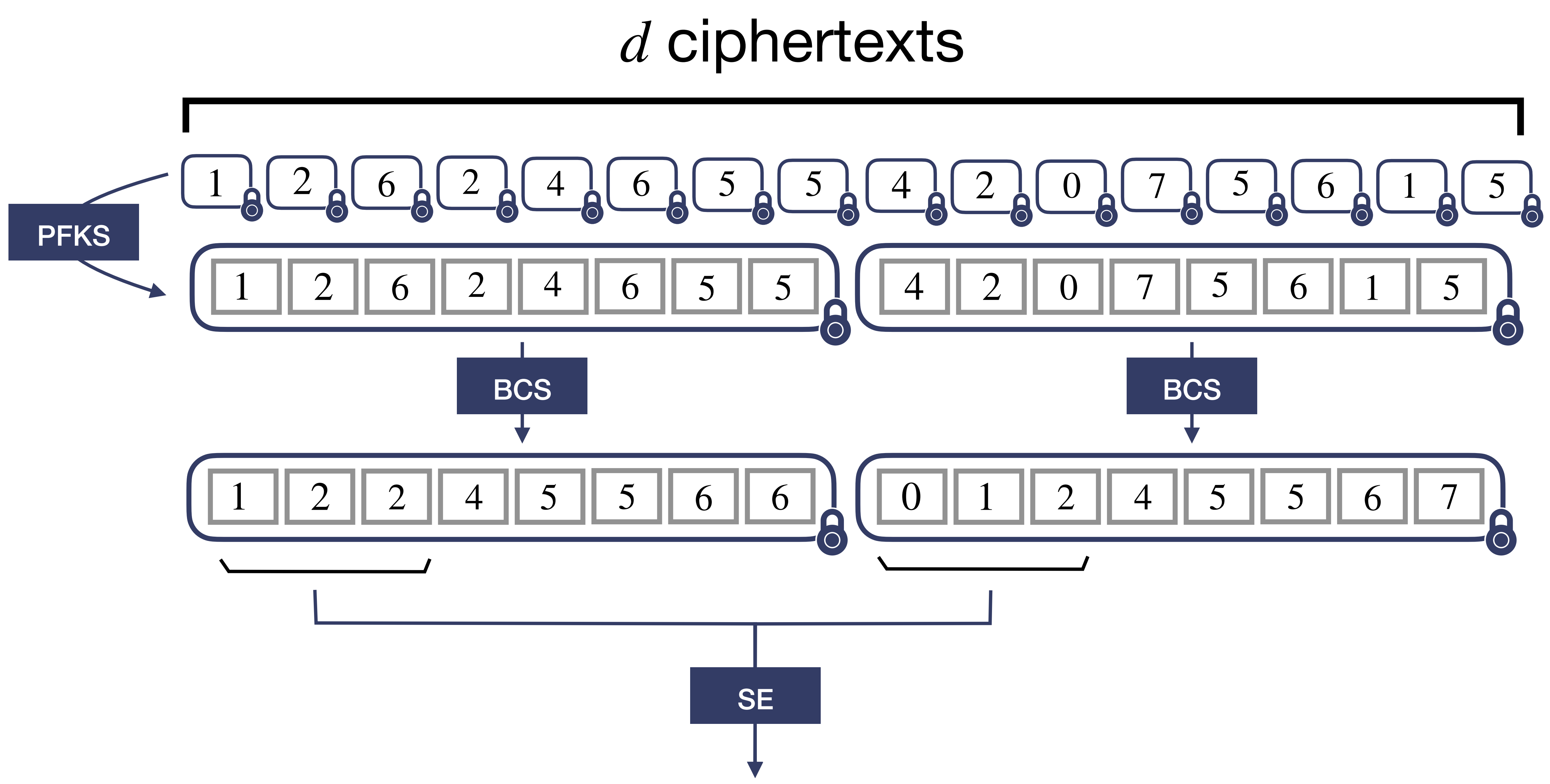

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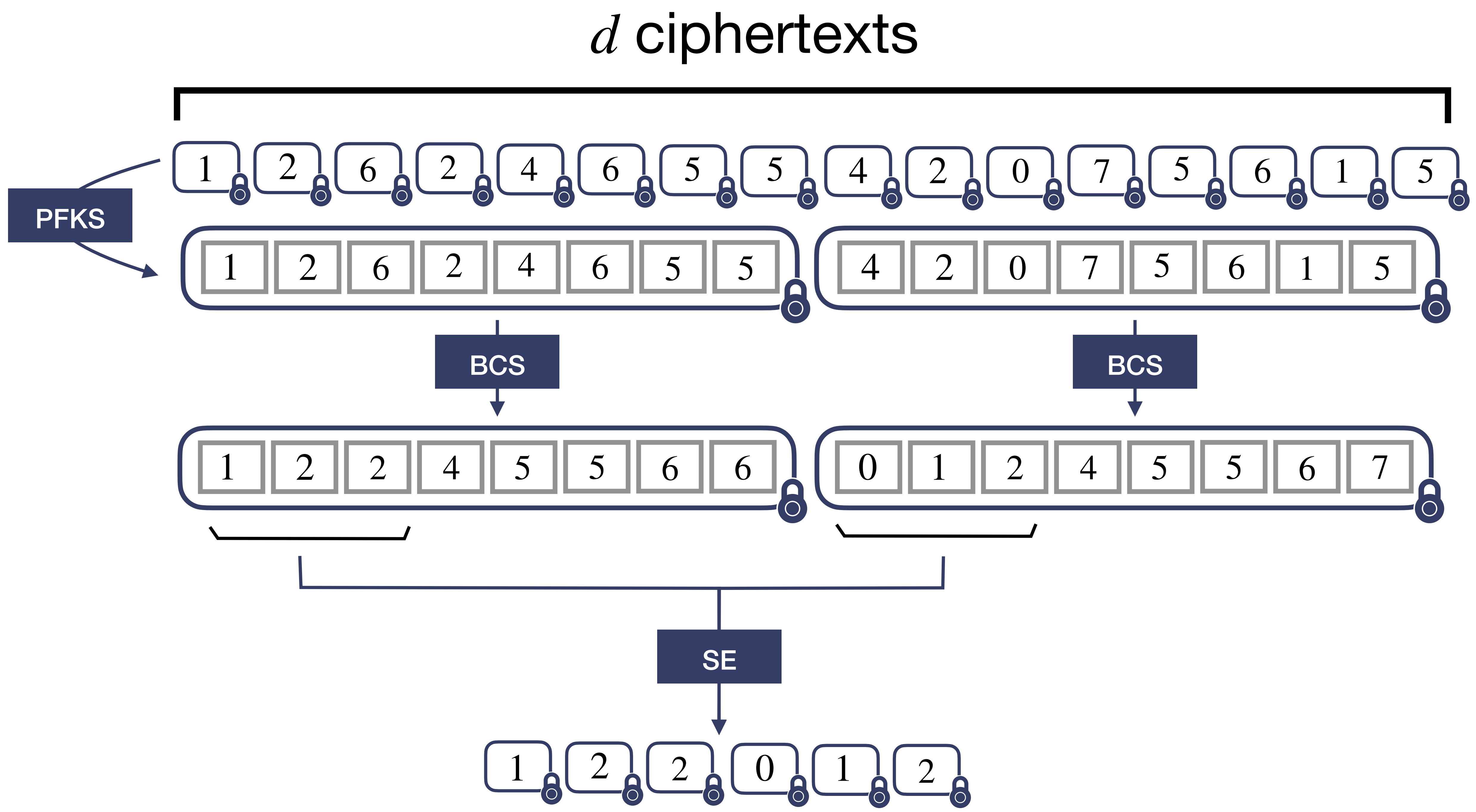

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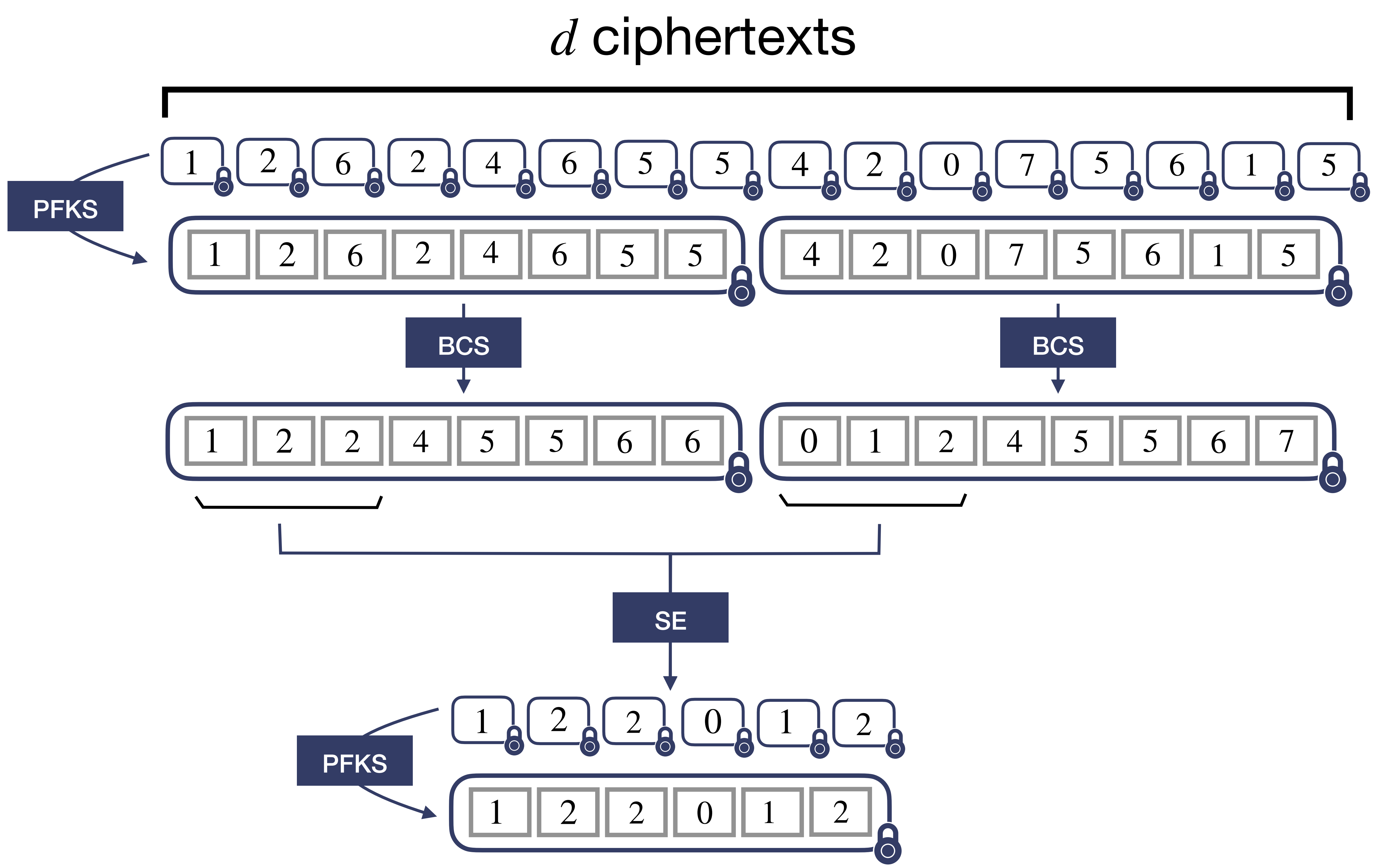

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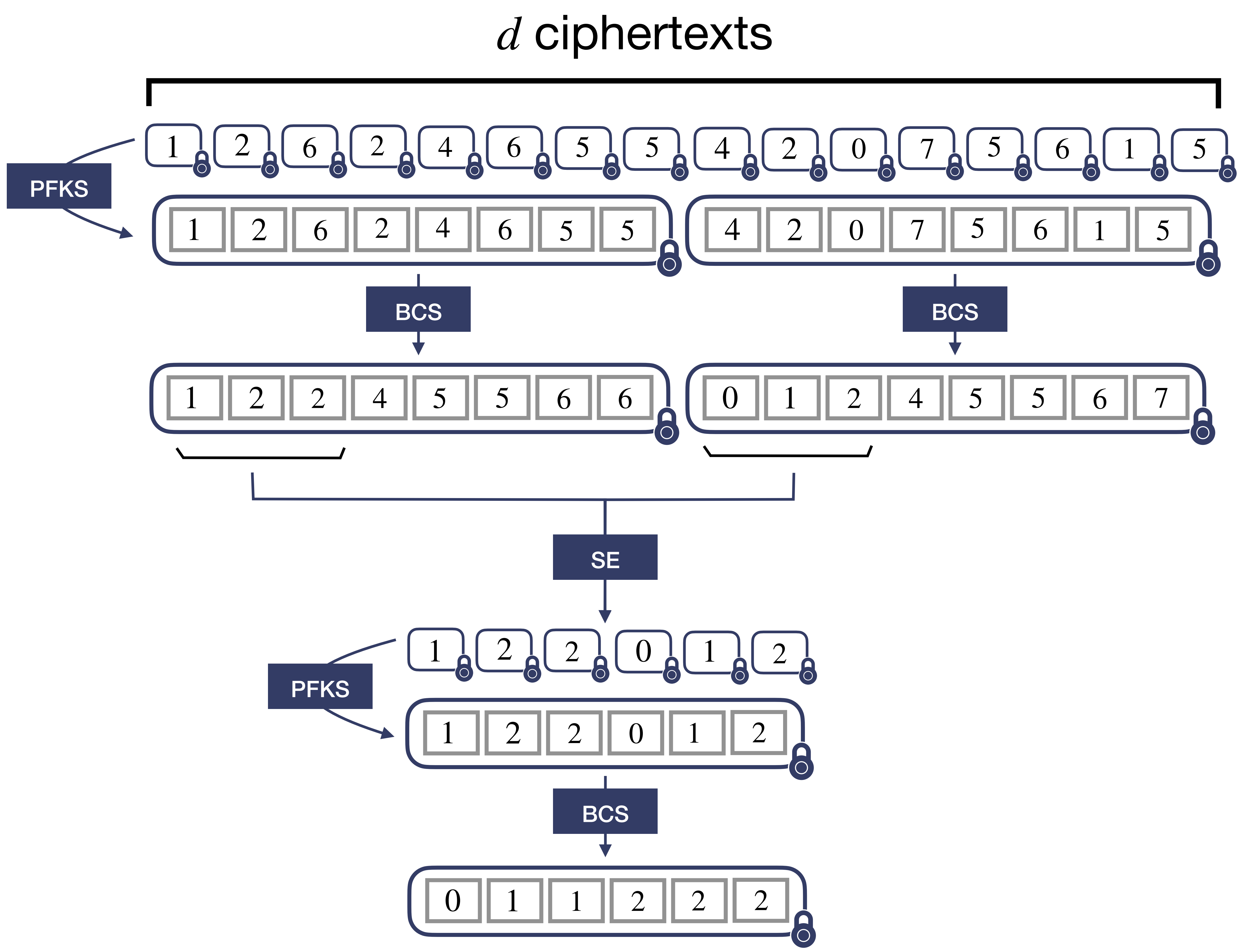

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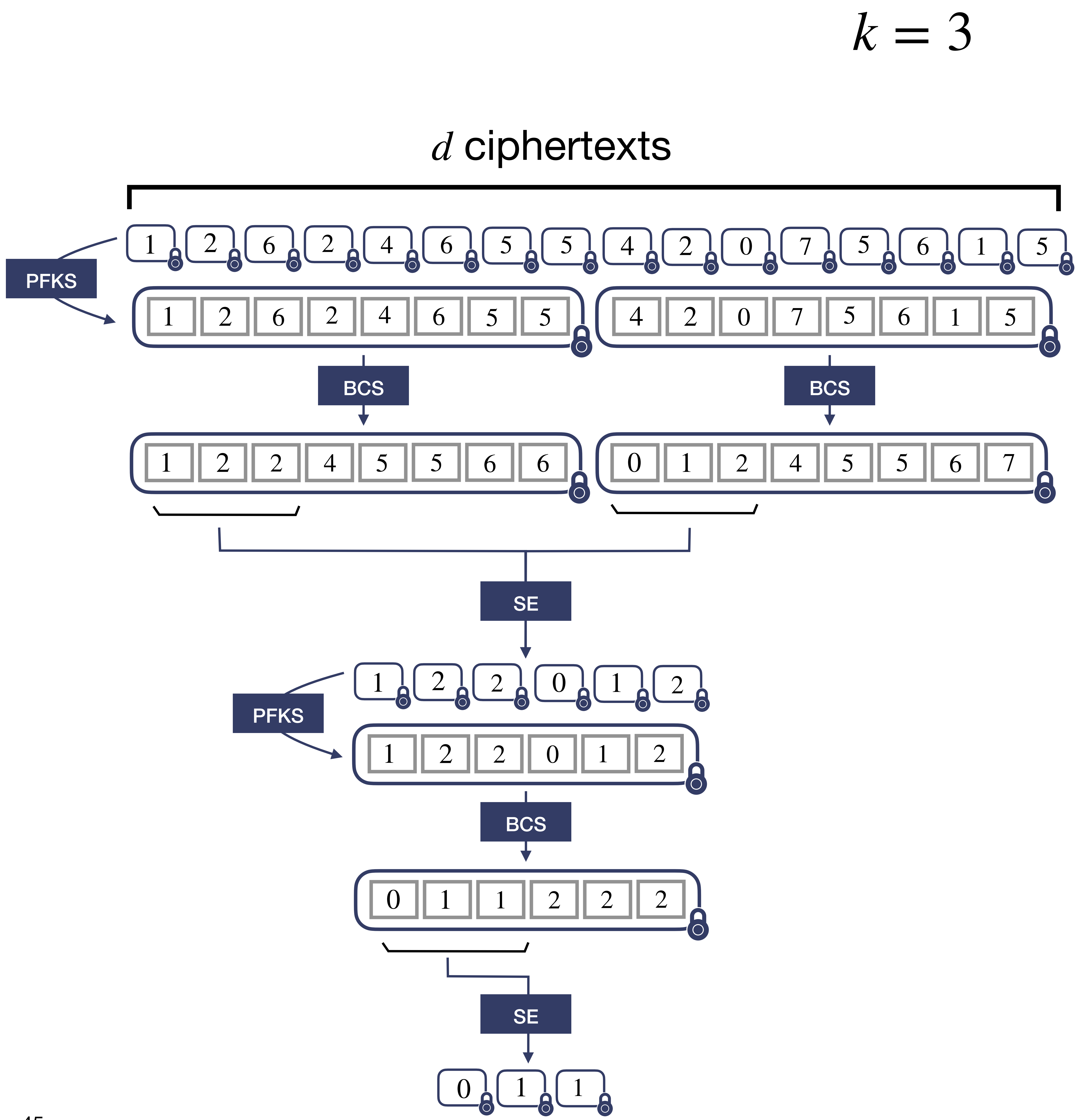


Blind Top-k

BCS used in a tournament

Blind Top-k

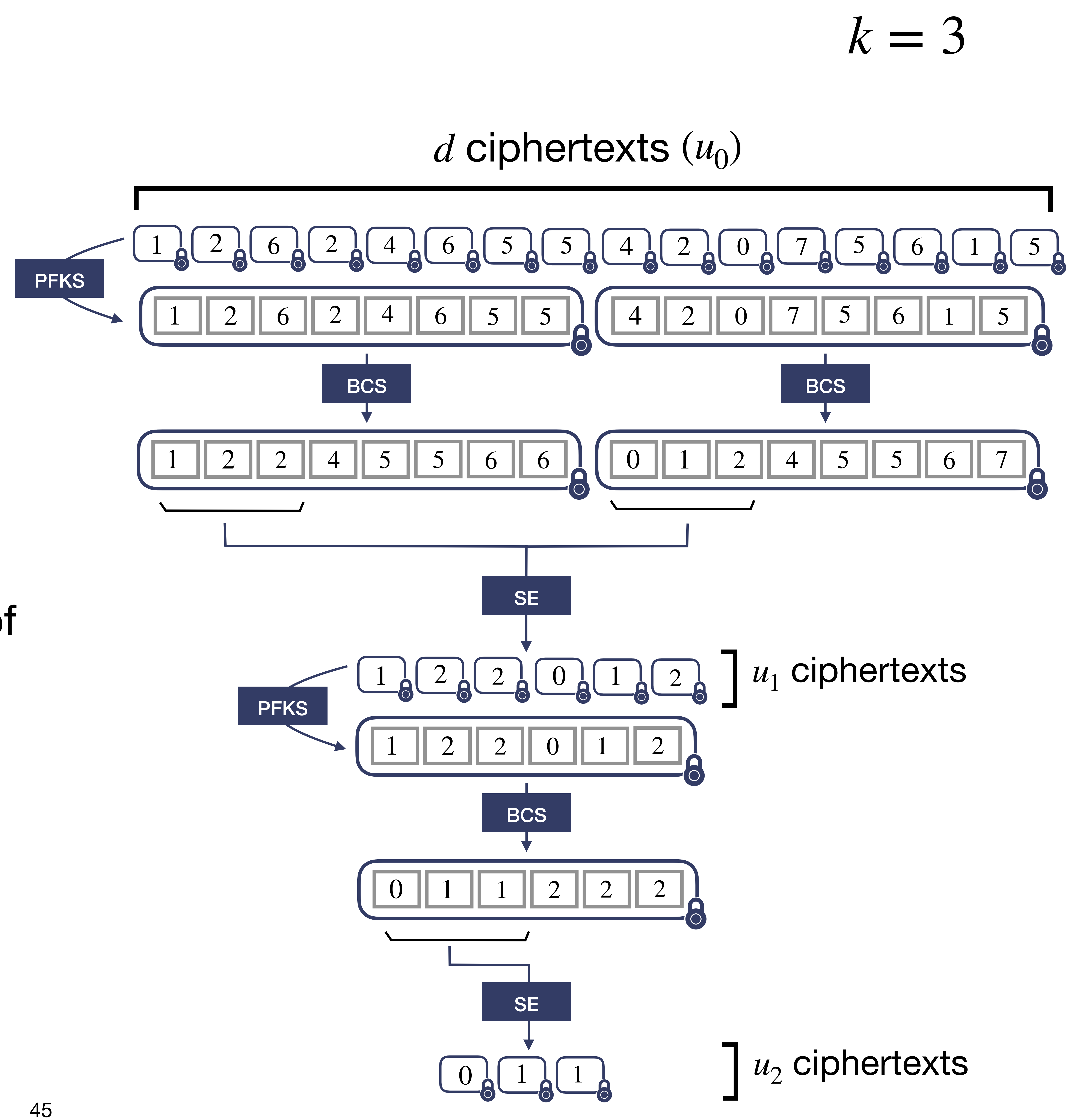
Total number of calls to BCS :

$$U = \left(\sum_{i=0}^{r-1} \left\lfloor \frac{u_i}{p} \right\rfloor \right) + 1$$

where r is the number of rounds and u_i the number of ciphertexts remaining after the i-th round.

Therefore, focusing on BR and PFKS, we have :

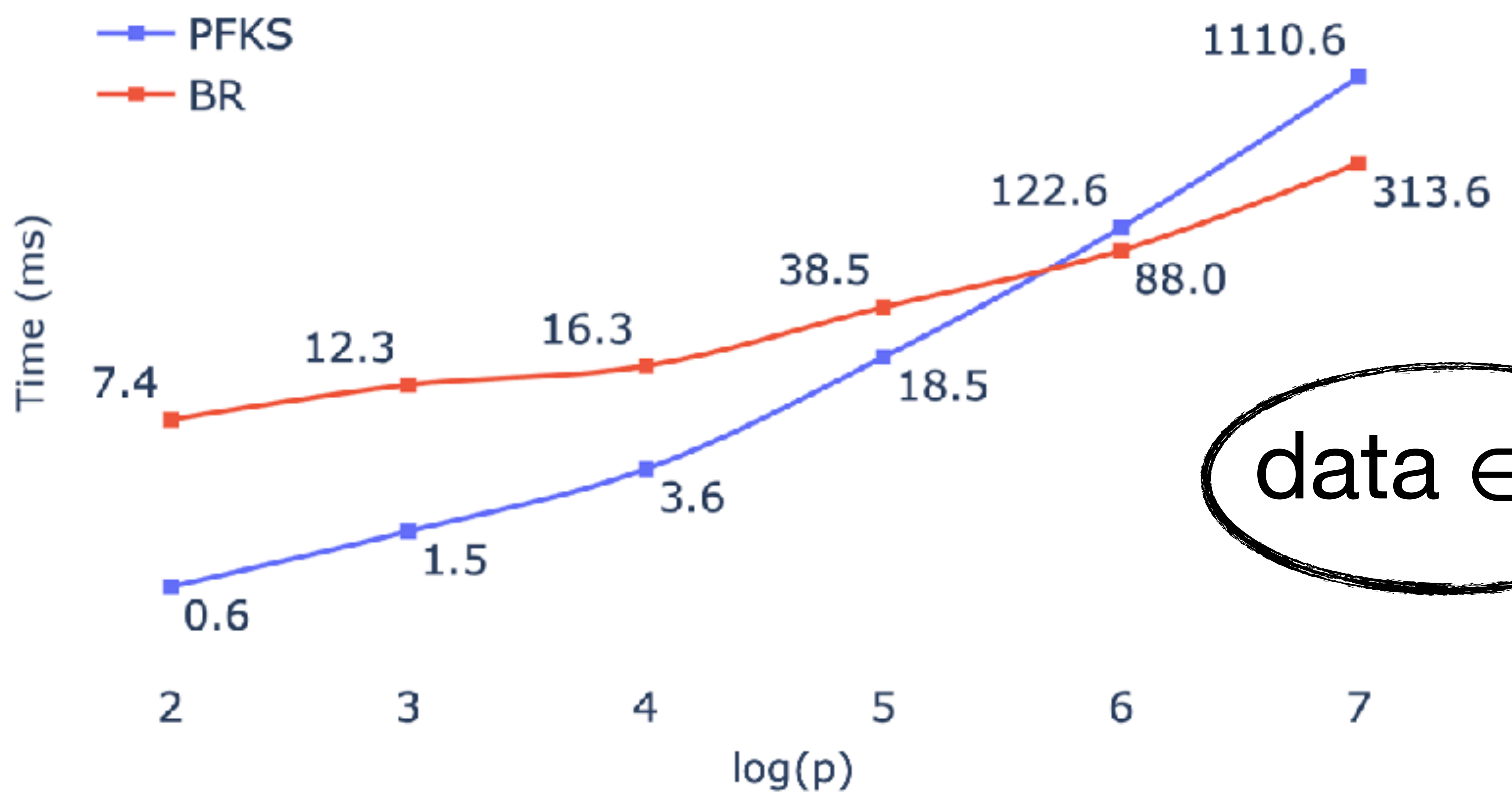
$$t_{BlindTopk} = 4pU \times \text{BR} + (2pU + r) \times \text{PFKS}$$



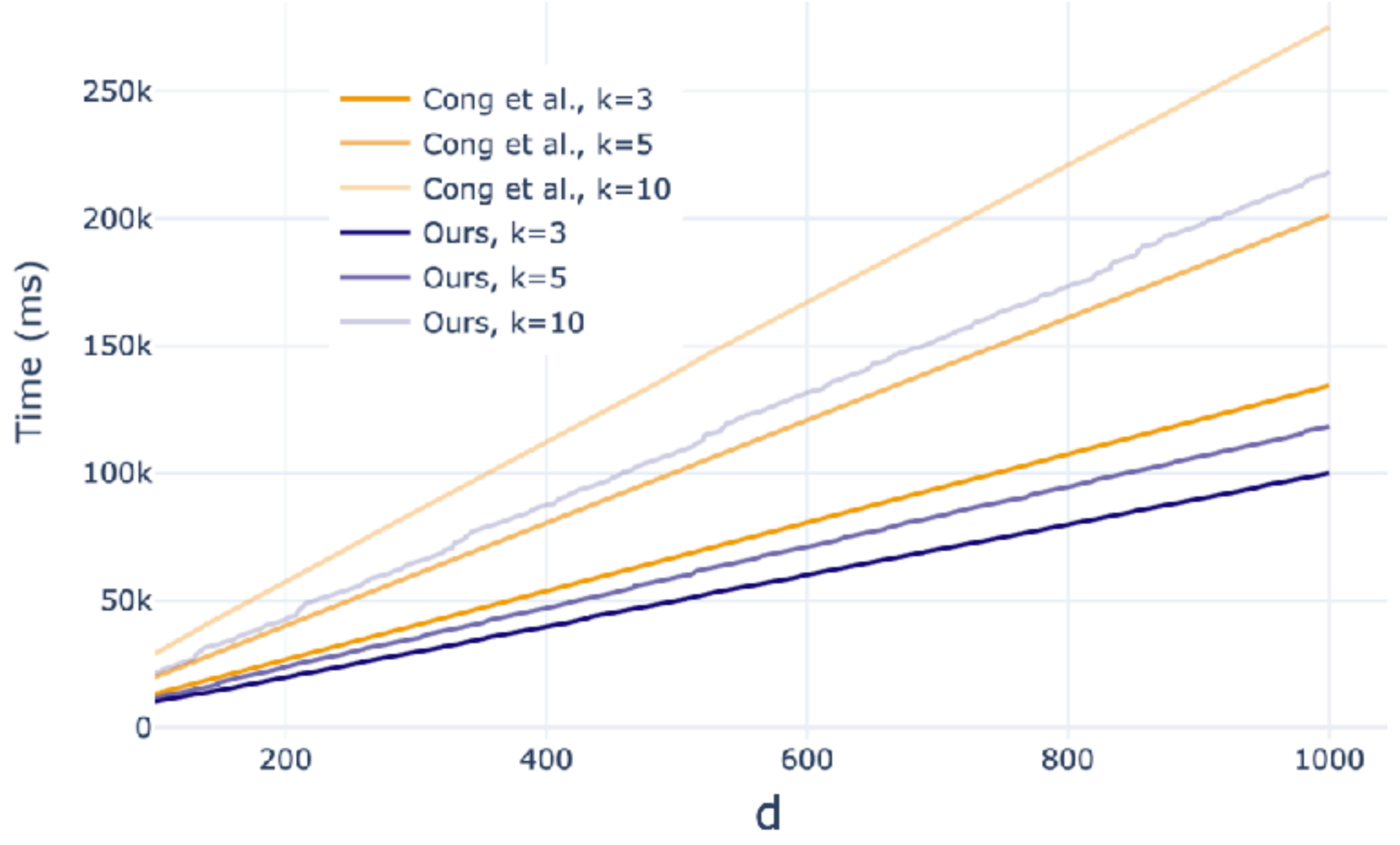
Blind Top-k

Comparison with the SOA

k	d	Cong et al.		Ours	
		BR	PFKS	BR	PFKS
3	40	186	372	190	148
	175	862	1724	995	668
	269	1332	2664	1565	1022
	457	2272	4544	2775	1794
	1000	4986	9972	6060	3846
5	40	250	500	210	156
	175	1196	2392	1215	810
	269	1856	3712	1860	1212
	457	3172	6344	3200	2054
	1000	6970	13940	7225	4582



data $\in \mathbb{Z}_{2^4}$



[Cong et al.] : Revisiting Oblivious Top-k Selection with Applications to Secure k-NN Classification, SAC 2024, LNCS 15516.

Blind Top-k

Comparison with the SOA

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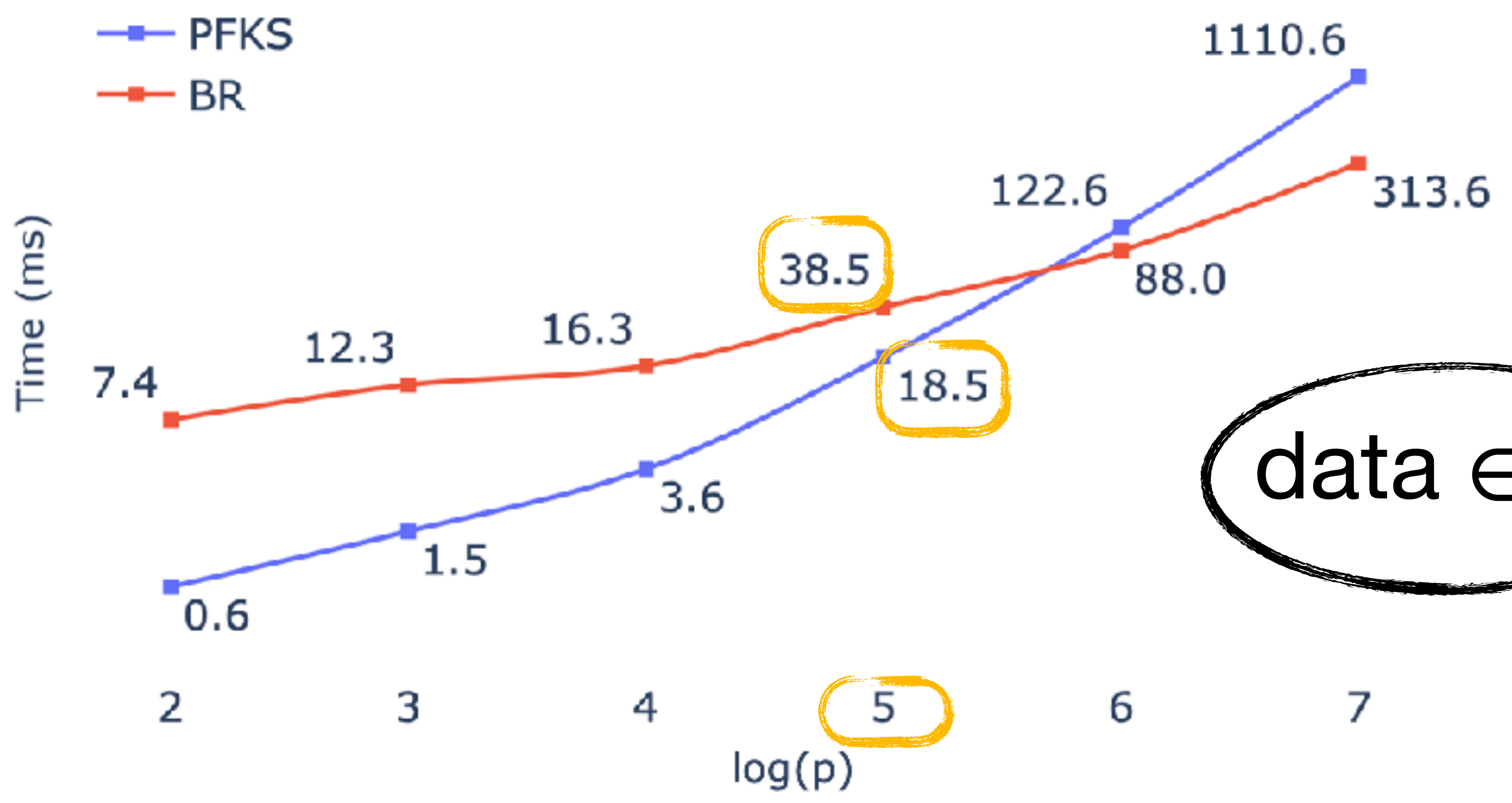
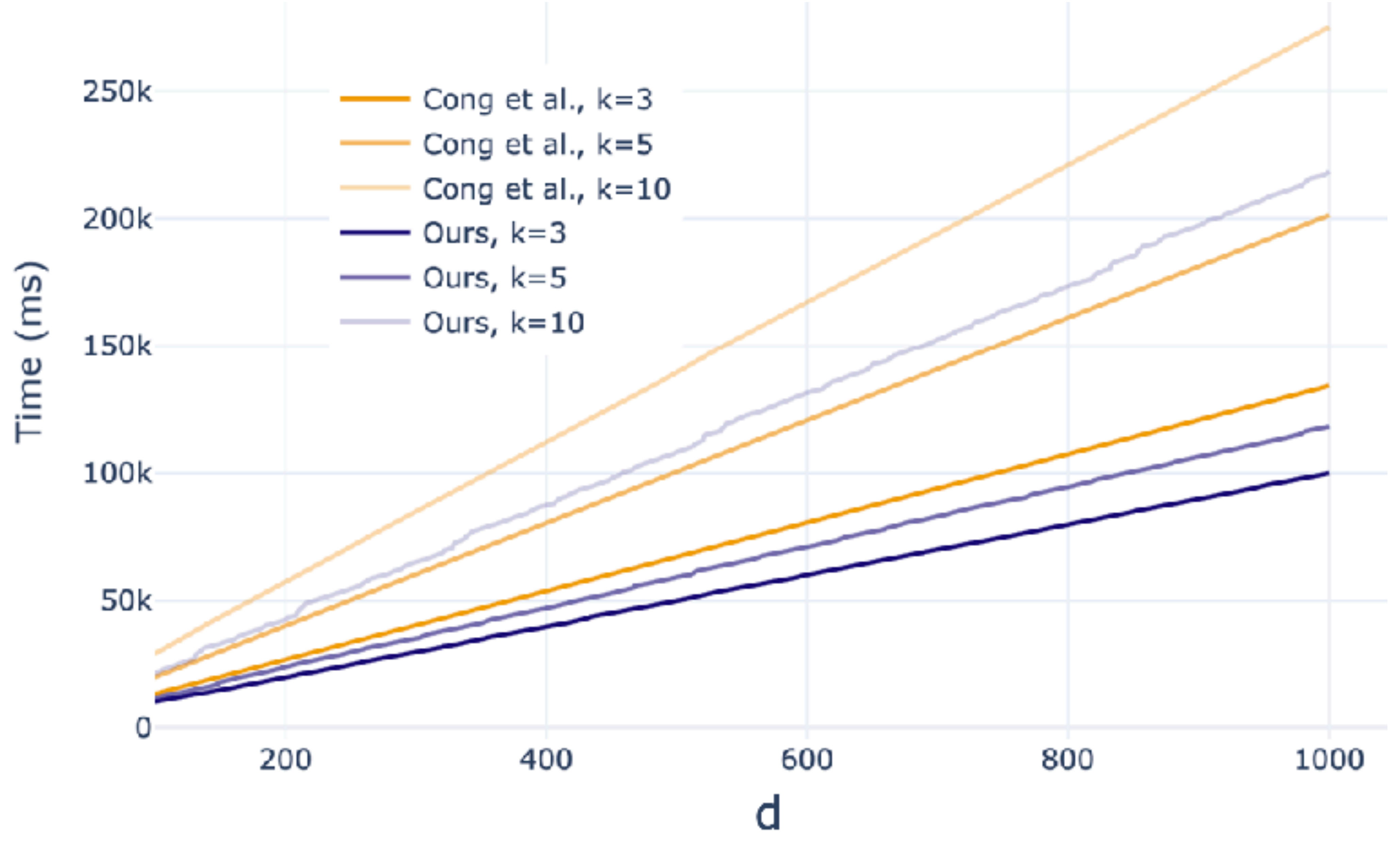


data $\in \mathbb{Z}_{24}$



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Blind Top-k

Comparison with the SOA

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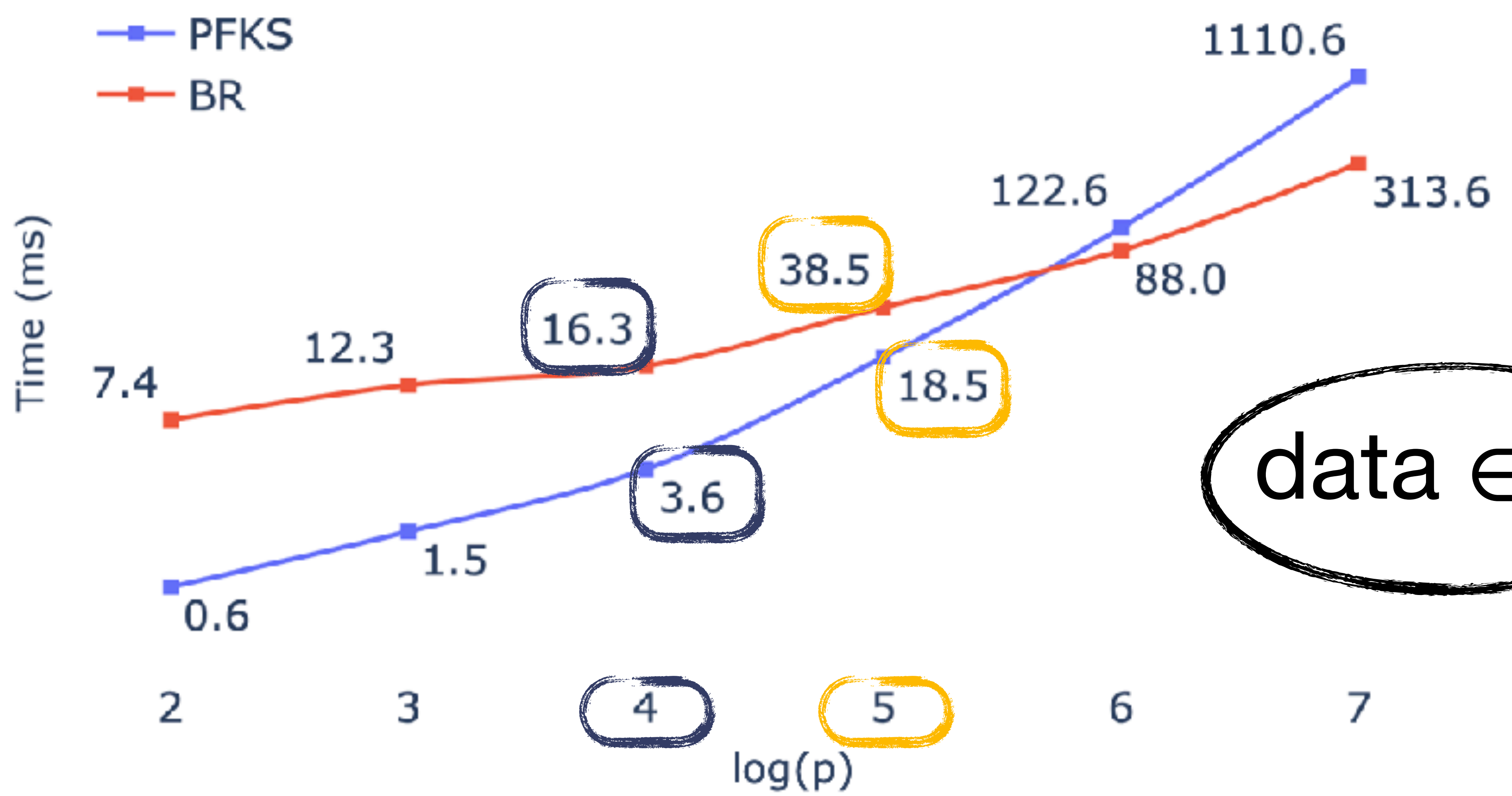
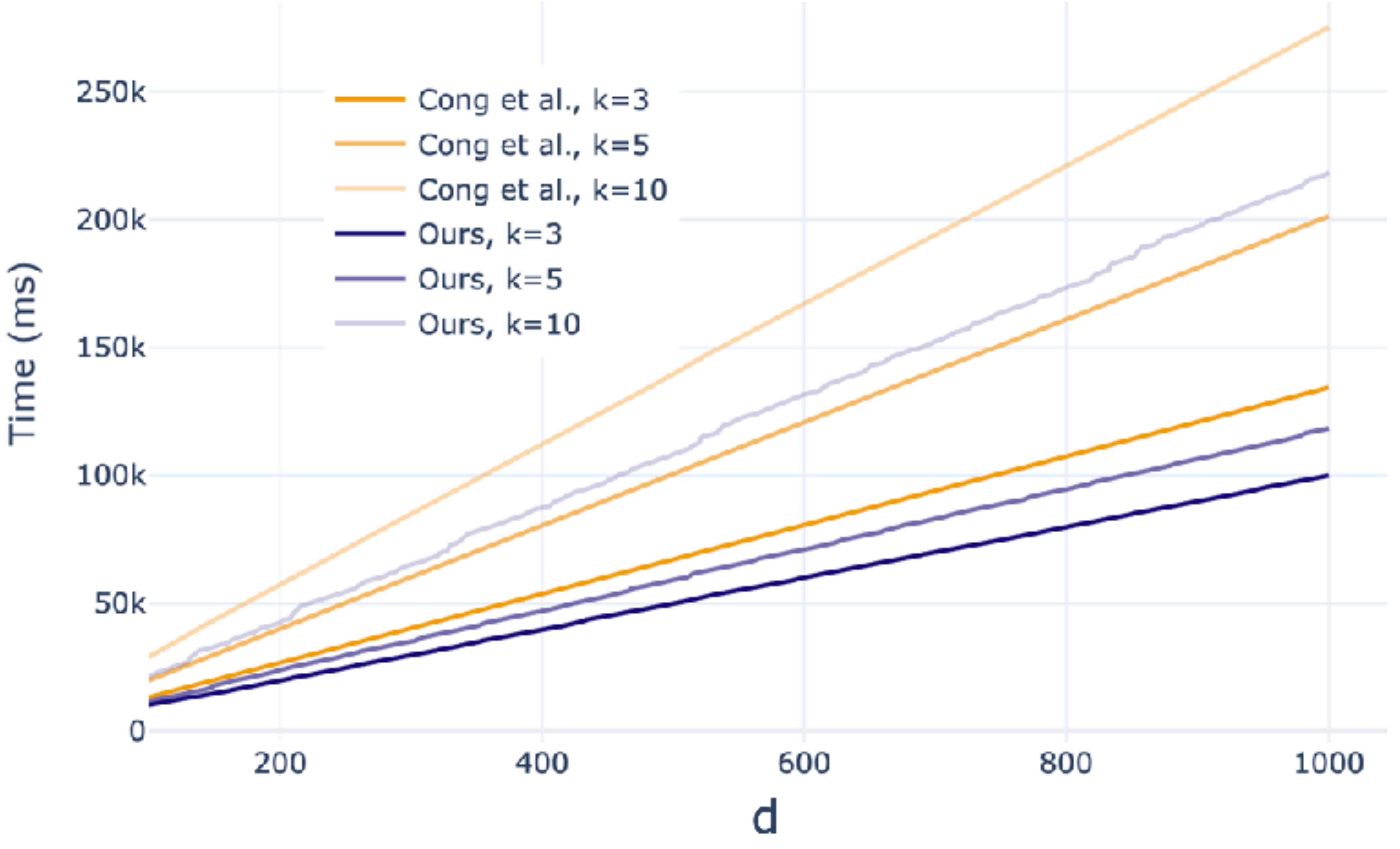


data $\in \mathbb{Z}_{24}$



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Private k-Nearest Neighbours

Private k-Nearest Neighbours

How it works in clear ?

Private k-Nearest Neighbours

How it works in clear ?

Client

Cloud

Private k-Nearest Neighbours

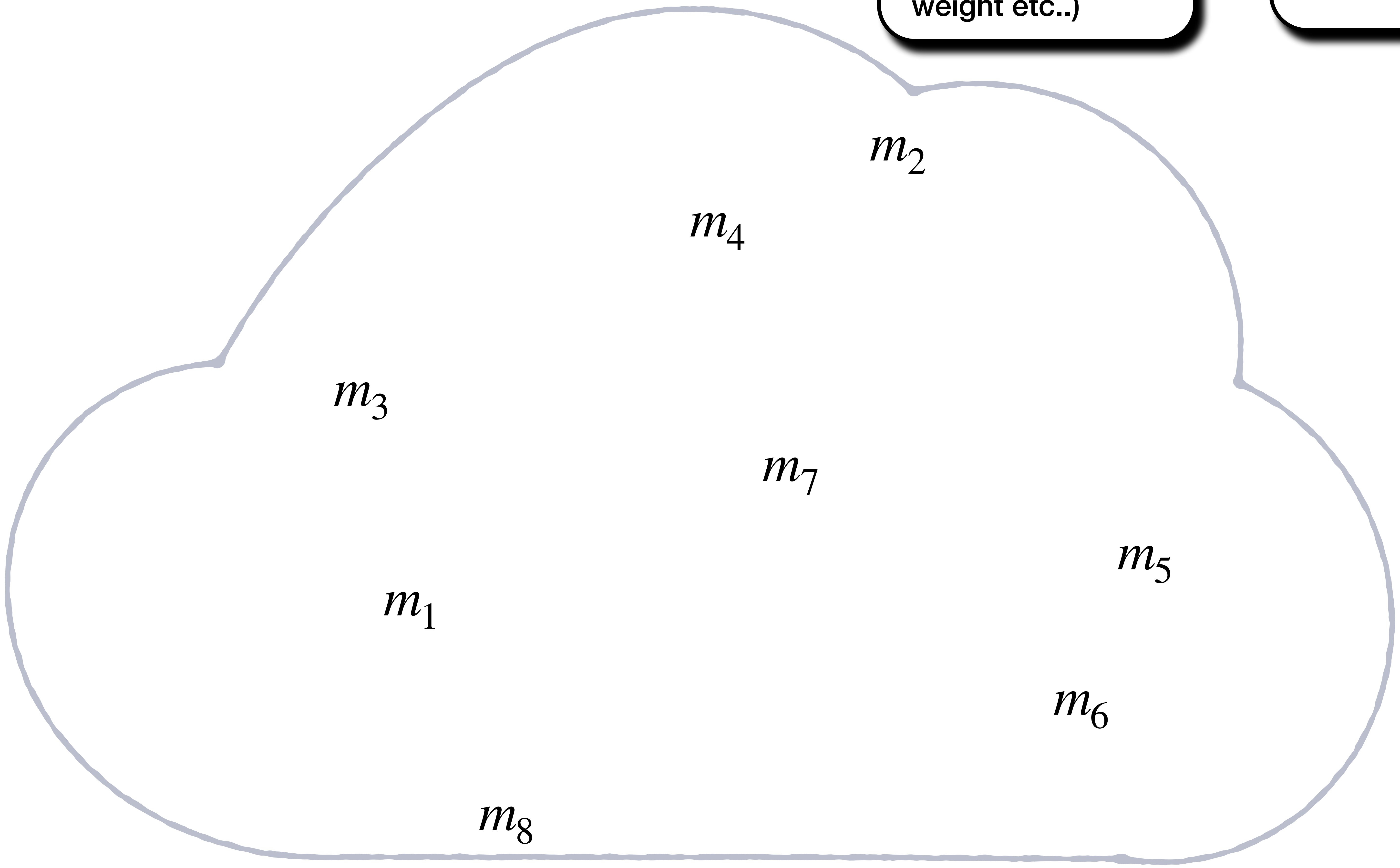
How it works in clear ?

$$m_i = (f_i, \ell_i)$$

Features (e.g age, weight etc..)

Label

Client



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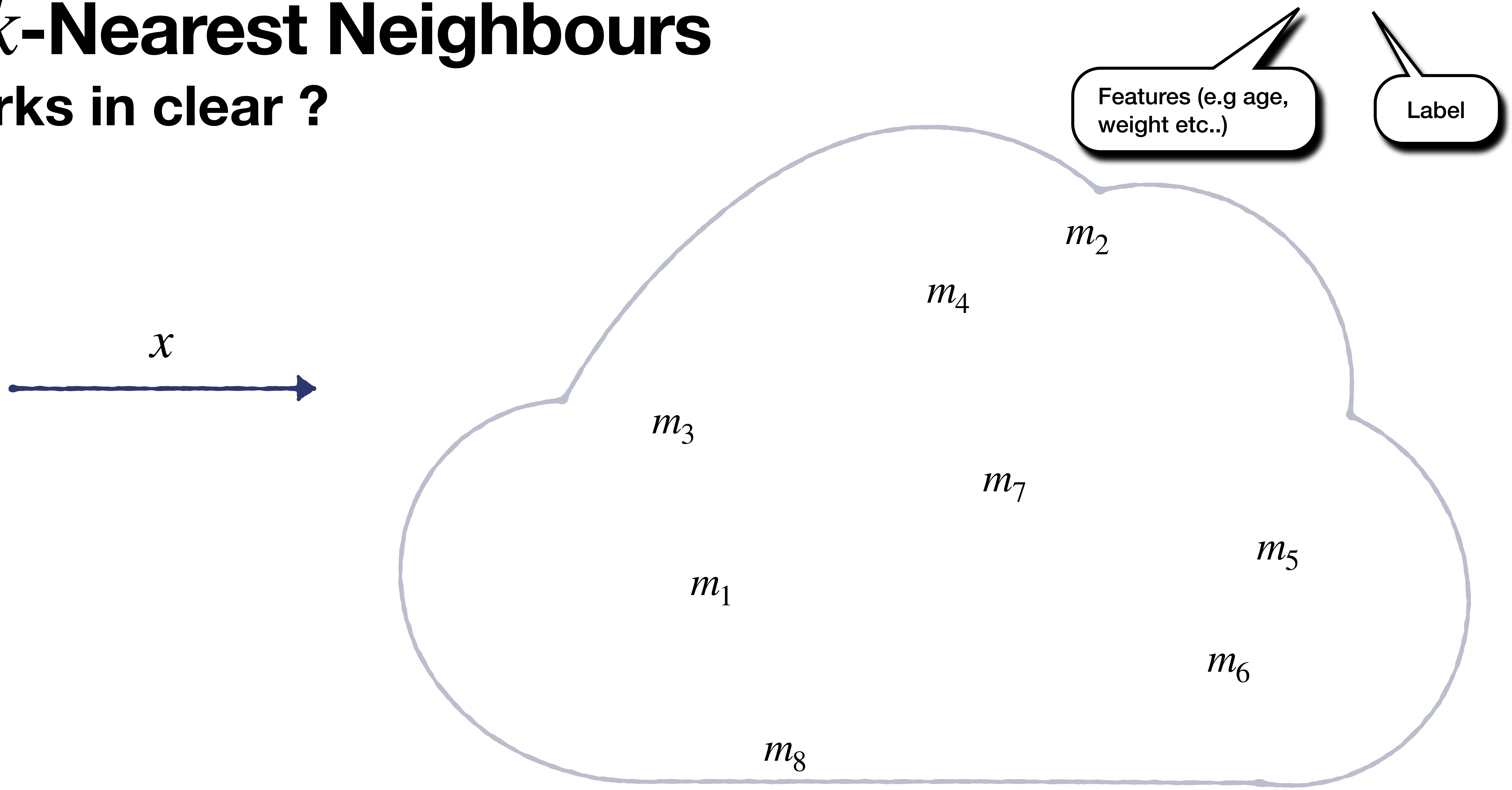

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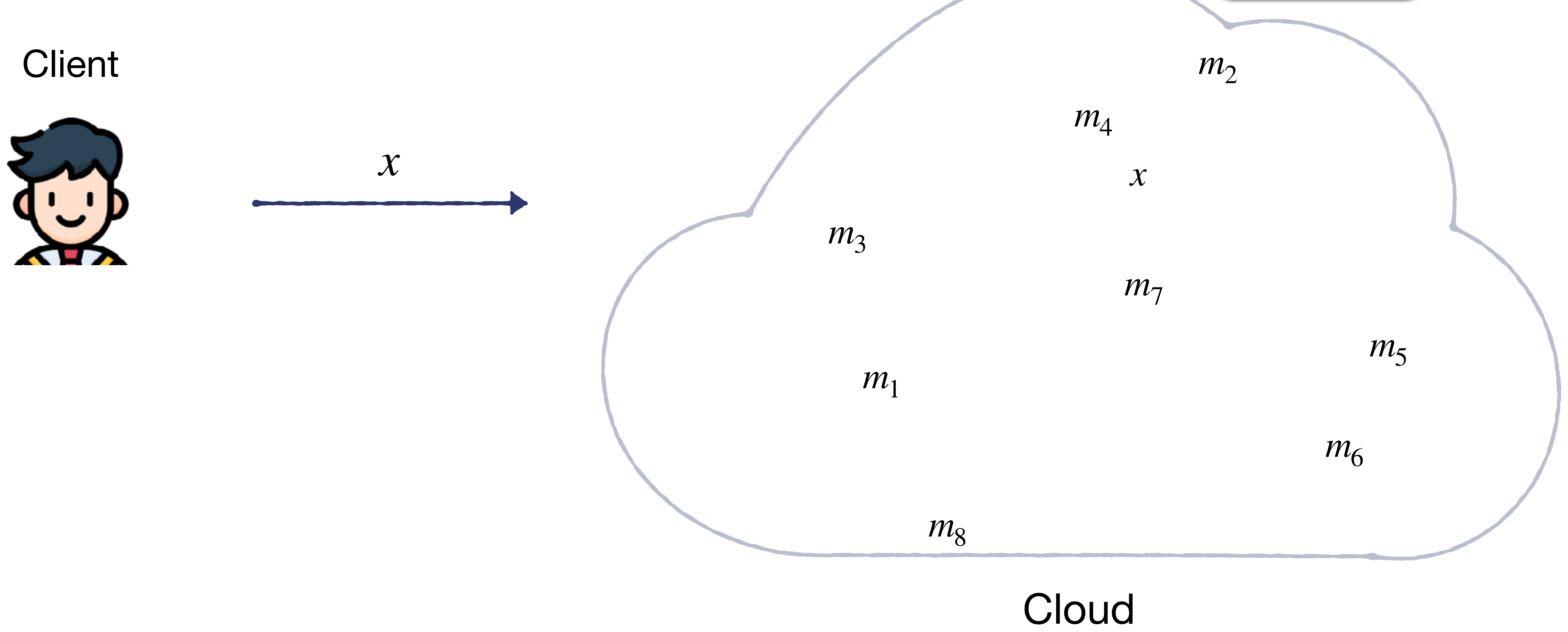

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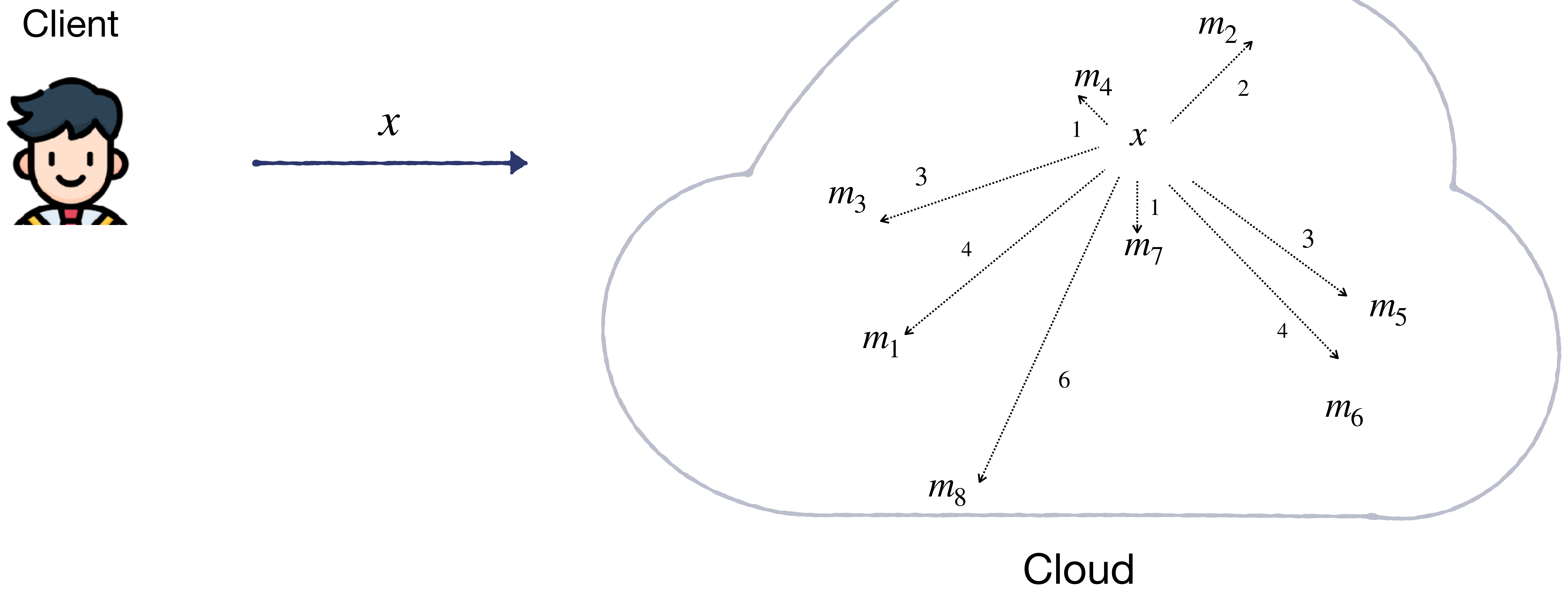

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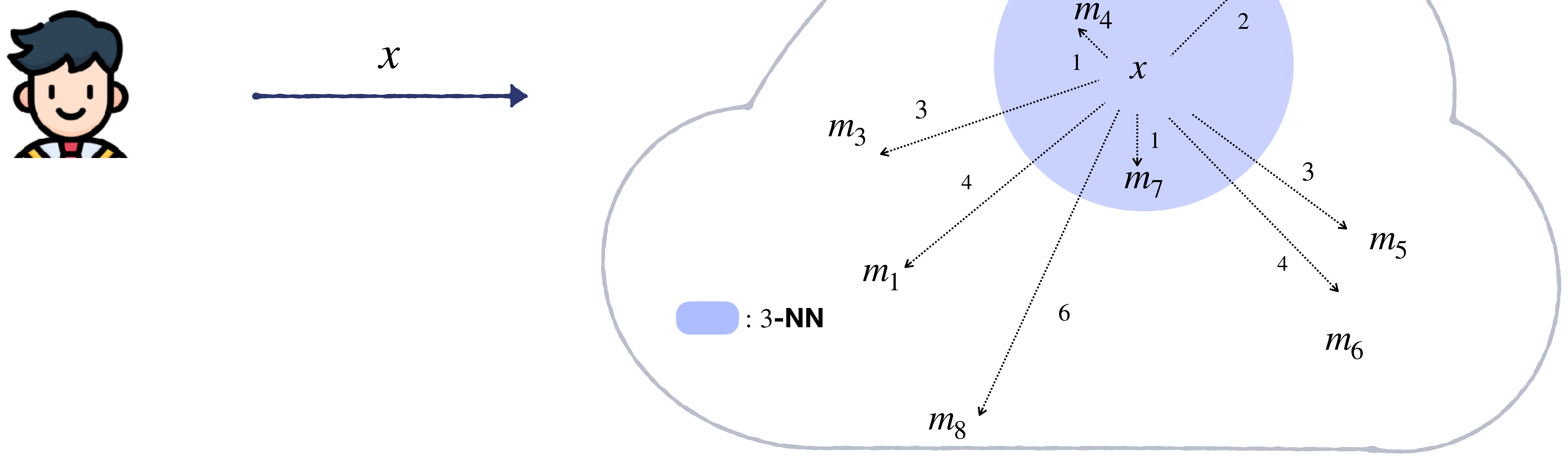

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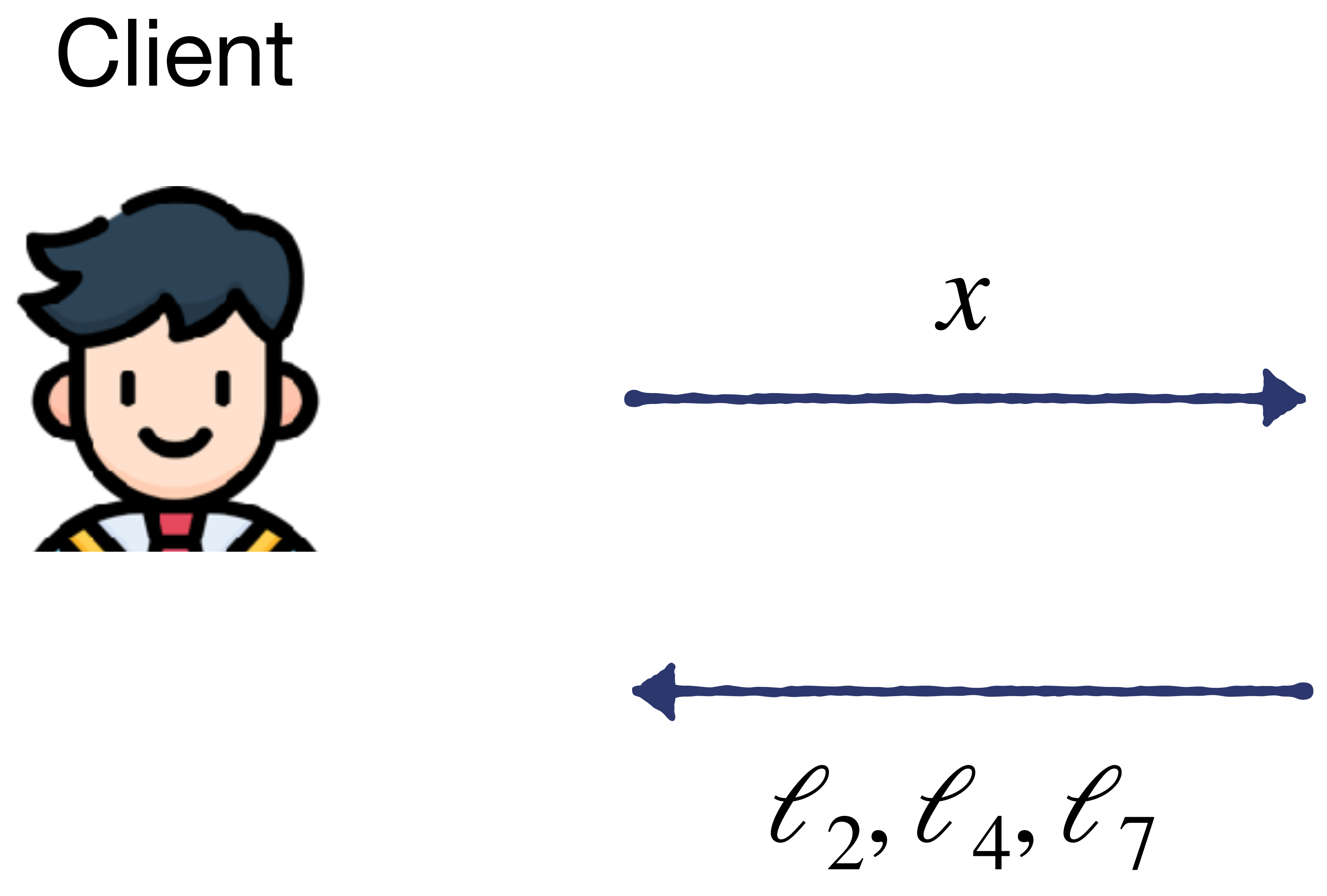
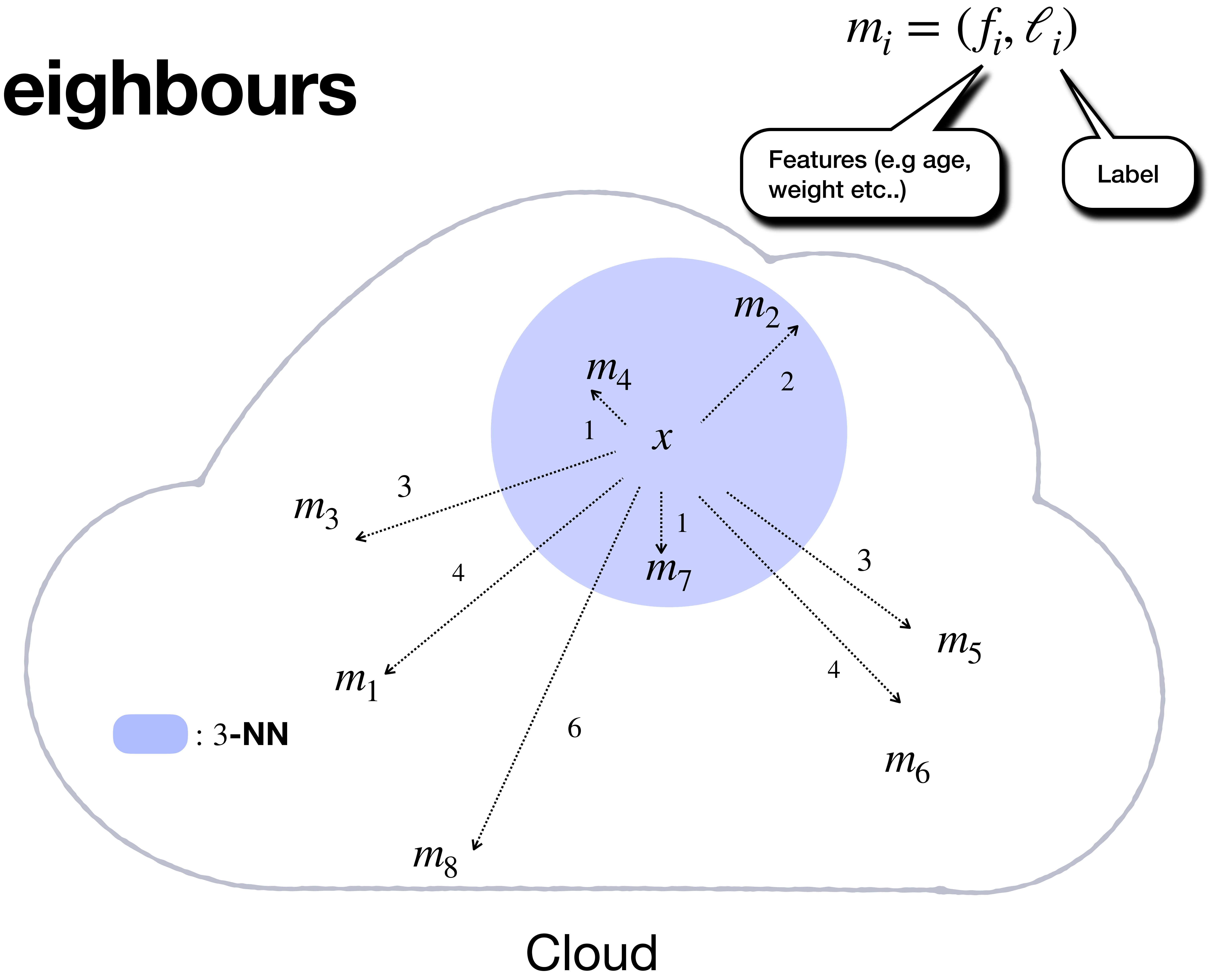




Private k-Nearest Neighbours

How it works in clear ?

$m_i = (f_i, \ell_i)$

Features (e.g age, weight etc..)

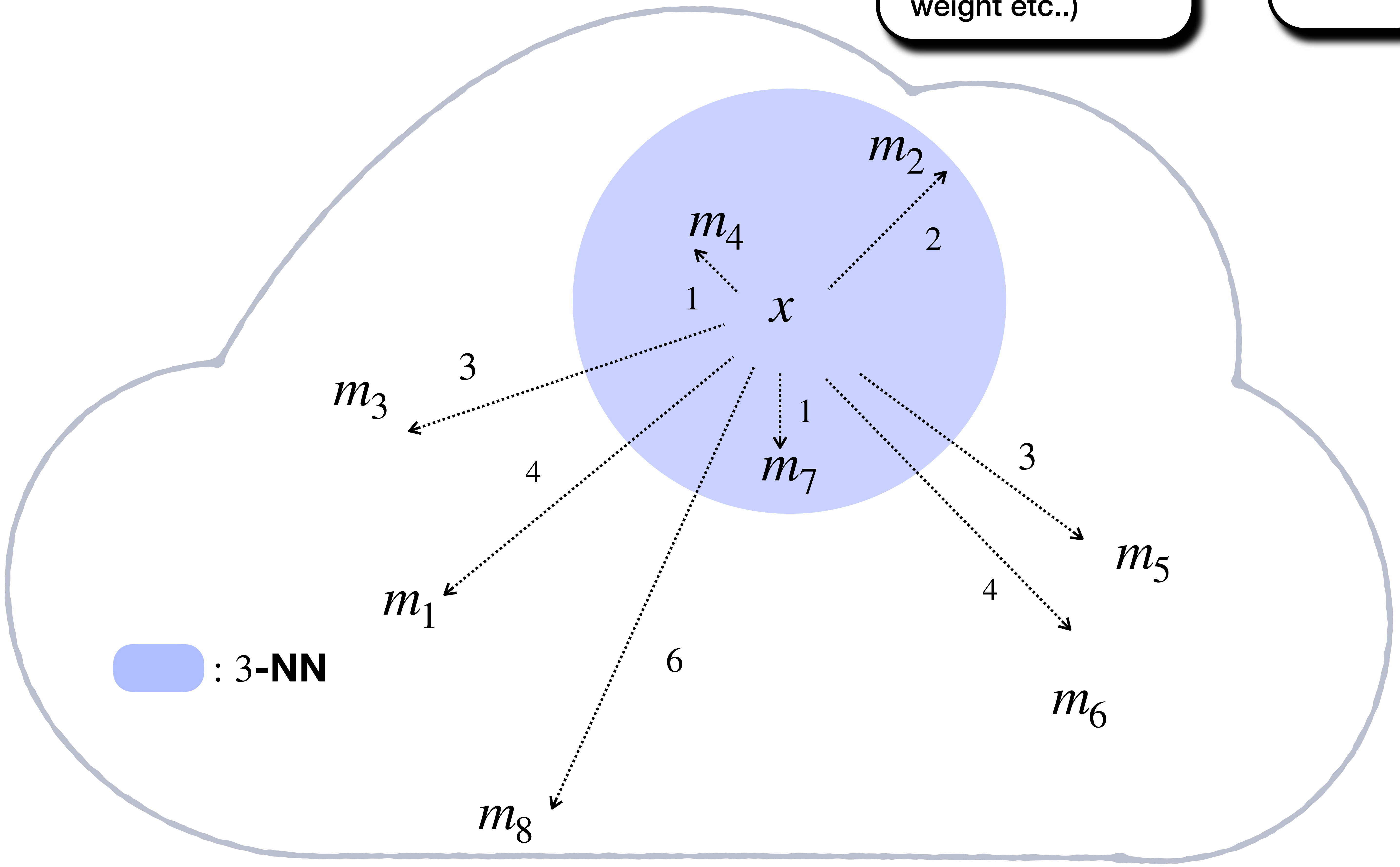
Label

Client

x

ℓ_2, ℓ_4, ℓ_7

$r \leftarrow$ Majority(ℓ_2, ℓ_4, ℓ_7)

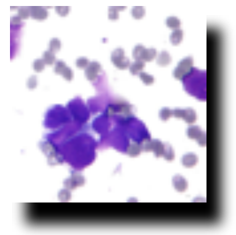


Cloud

Private k-Nearest Neighbours

Datasets used

$p = 2^4$



Dataset	Dimension	Message space	Dataset size	p_{dist}
Breast Cancer	30	$\mathbb{Z}_2$	569	2^4
MNIST	64	$\mathbb{Z}_2$	1797	2^5

A precision reduction is needed for the MNIST dataset as $p \leq p_{dist}$

Private k-Nearest Neighbours

Distances computation in $\mathbb{Z}_{p_{dist}}$

m_2

m_4

m_3

m_7

m_5

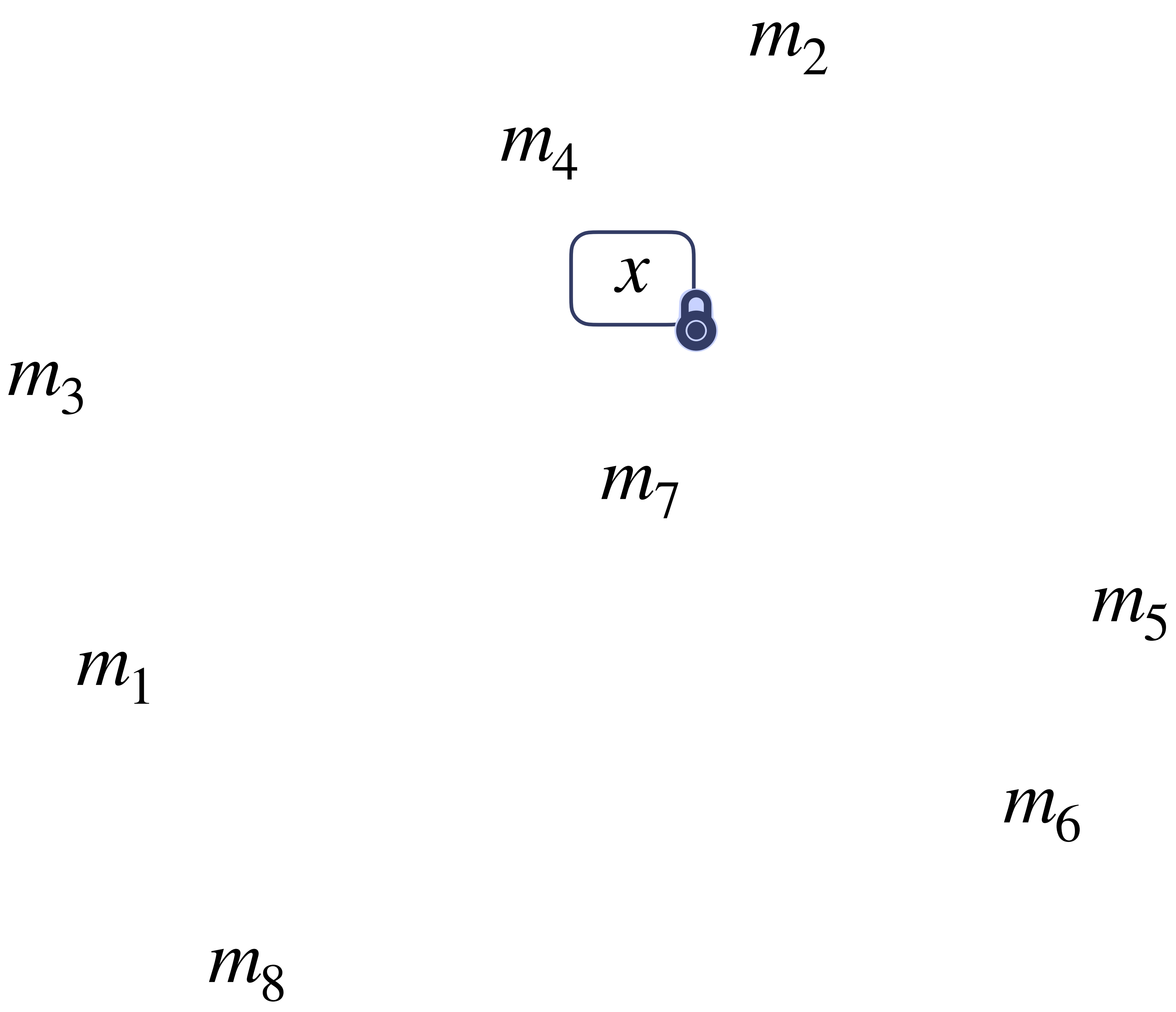
m_1

m_6

m_8

Private k-Nearest Neighbours

Distances computation in $\mathbb{Z}_{p_{dist}}$



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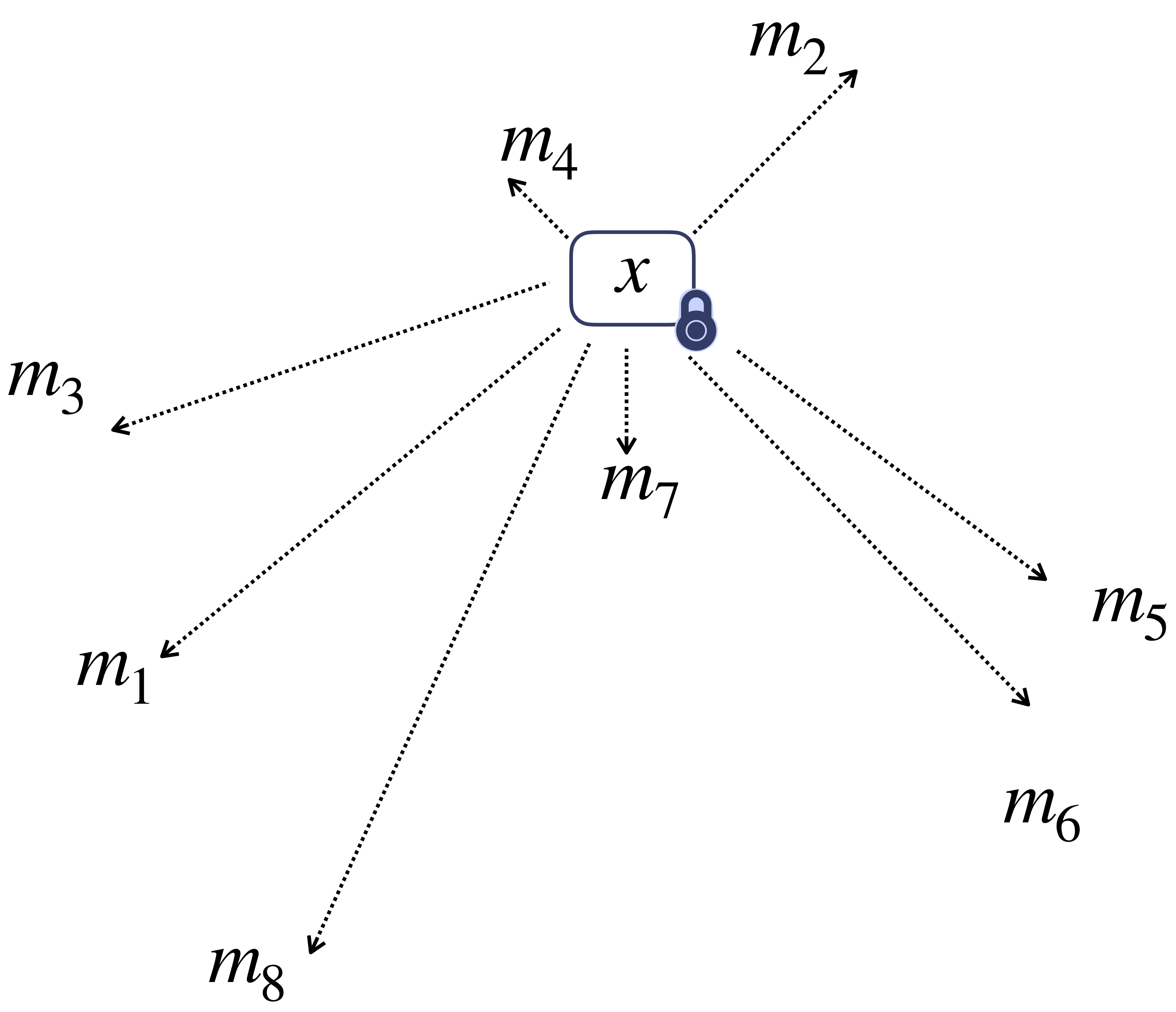

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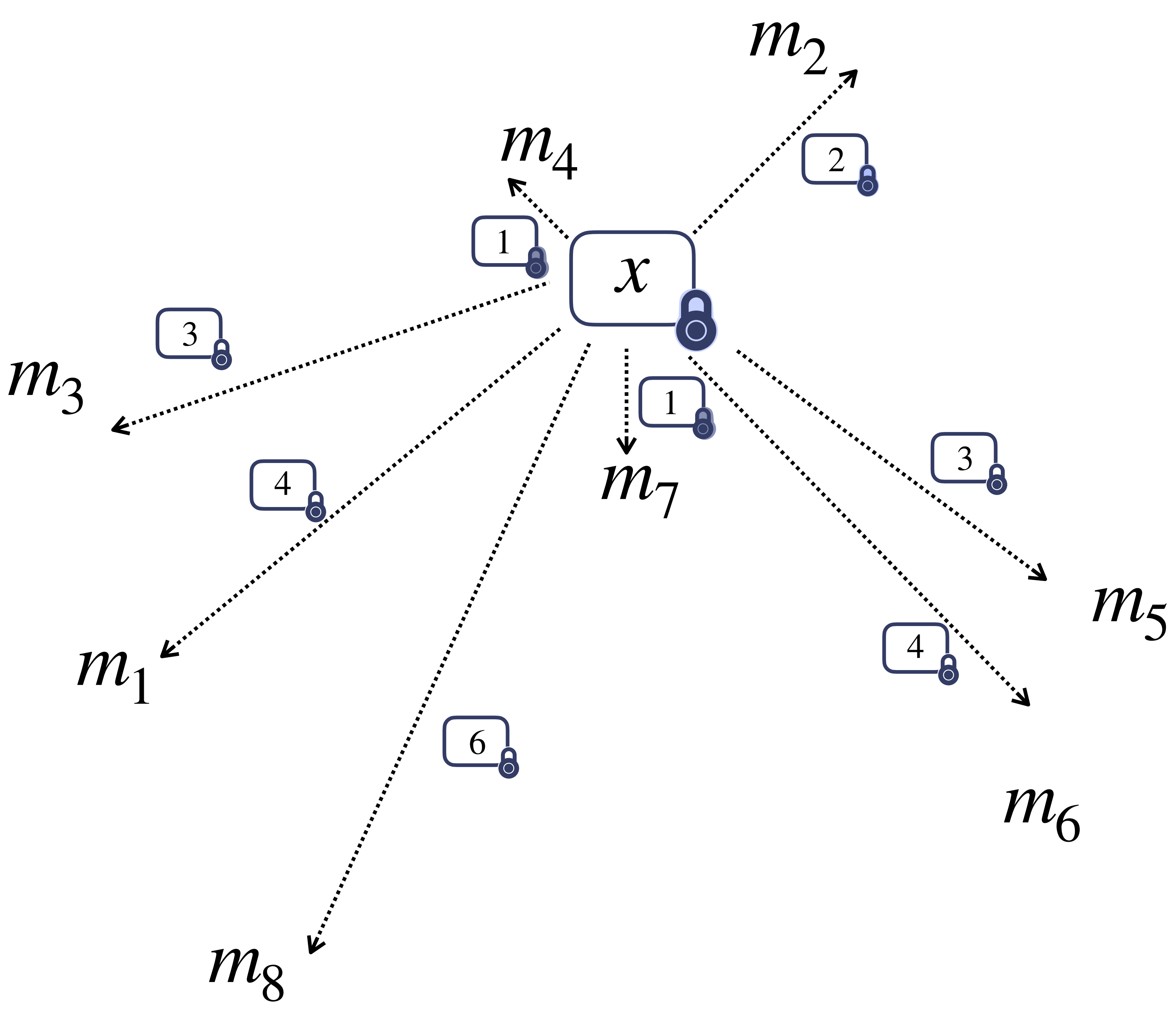
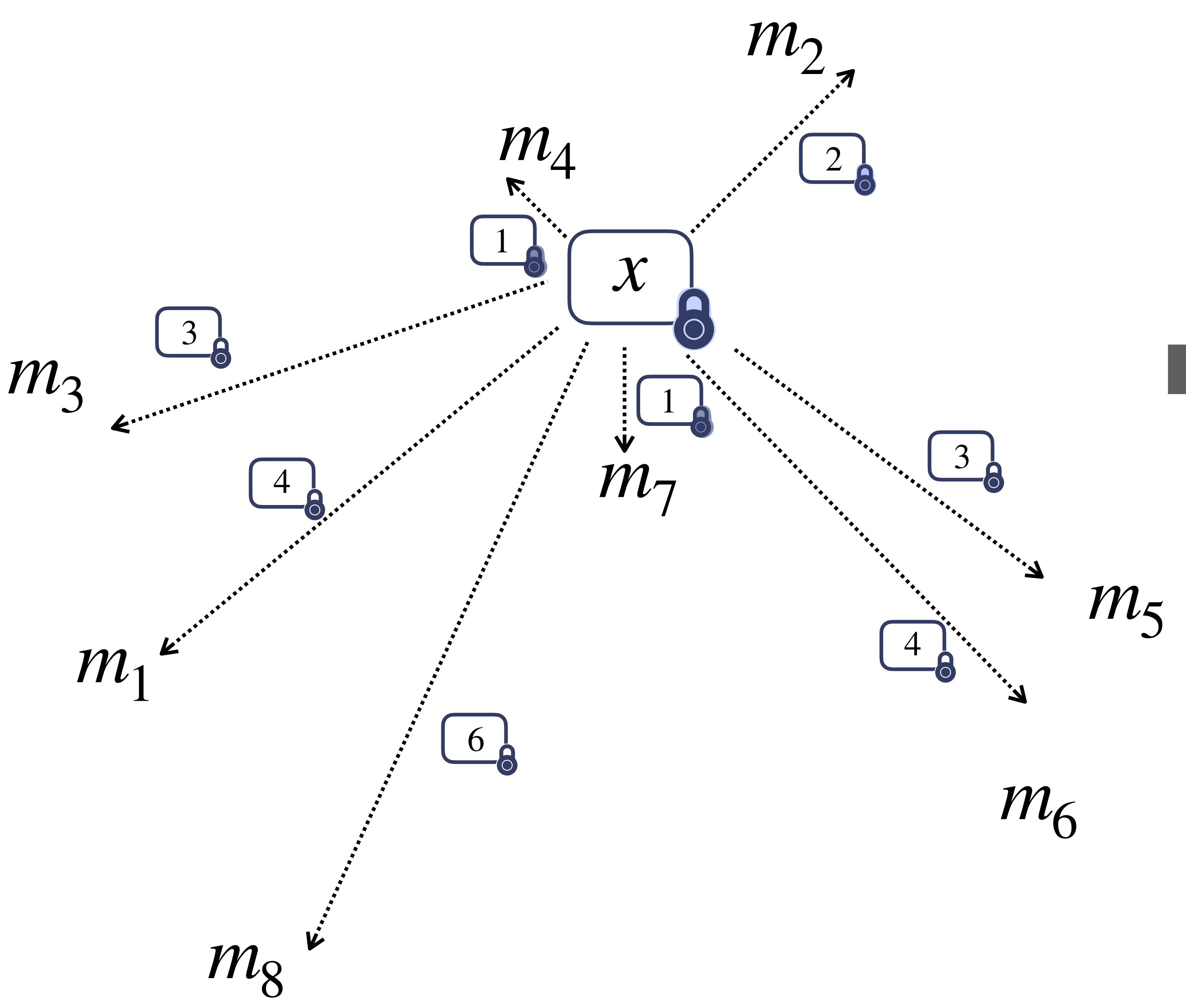


Private k-Nearest Neighbours

Distances computation in $\mathbb{Z}_{p_{dist}}$

Precision reduction (if needed)



Distances plaintext from $\mathbb{Z}_{p_{dist}}$ to $\mathbb{Z}_p$

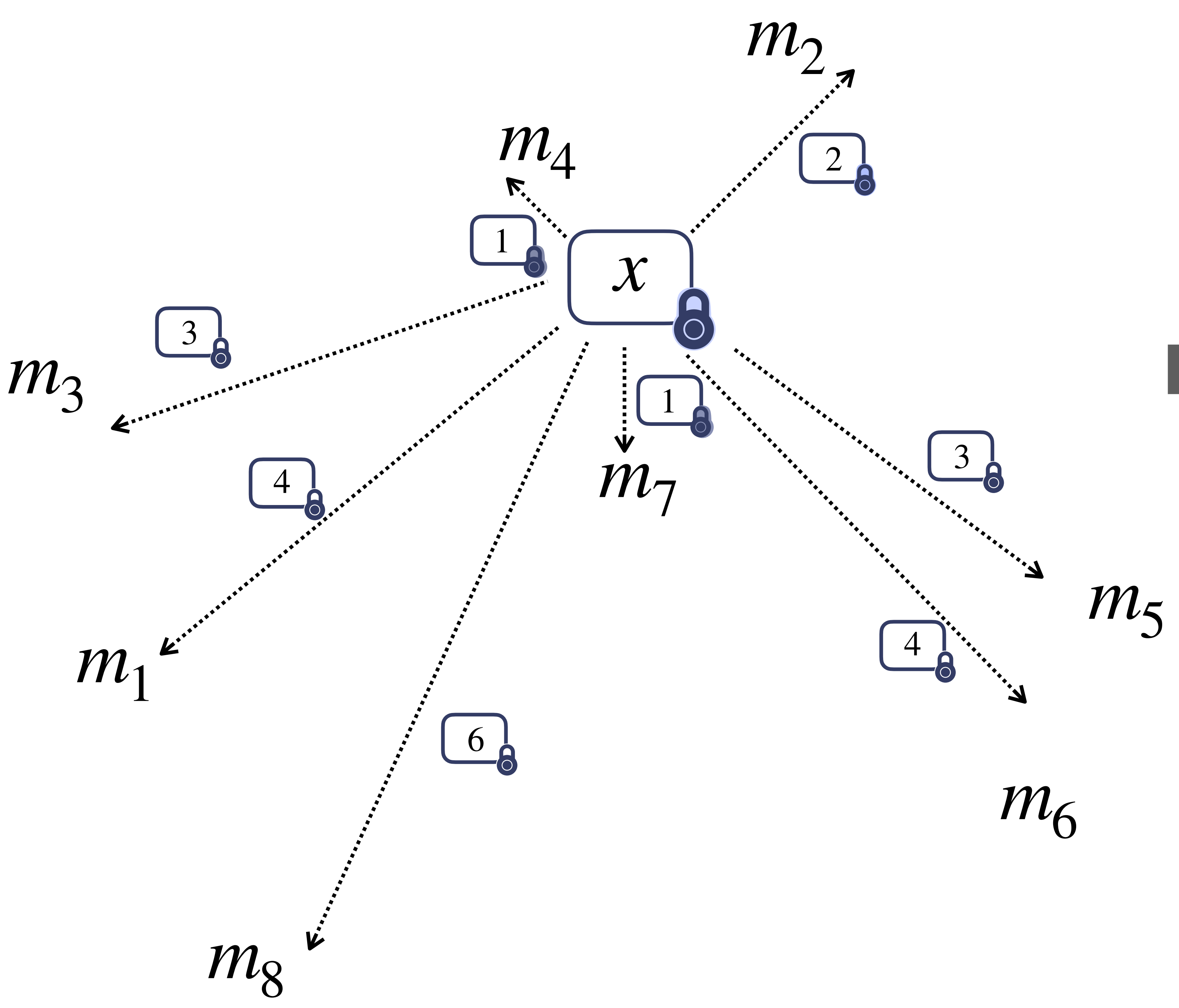
May introduce additional noise

Private k-Nearest Neighbours

Distances computation in $\mathbb{Z}_{p_{dist}}$

Precision reduction (if needed)

Top-k selection in $\mathbb{Z}_p$



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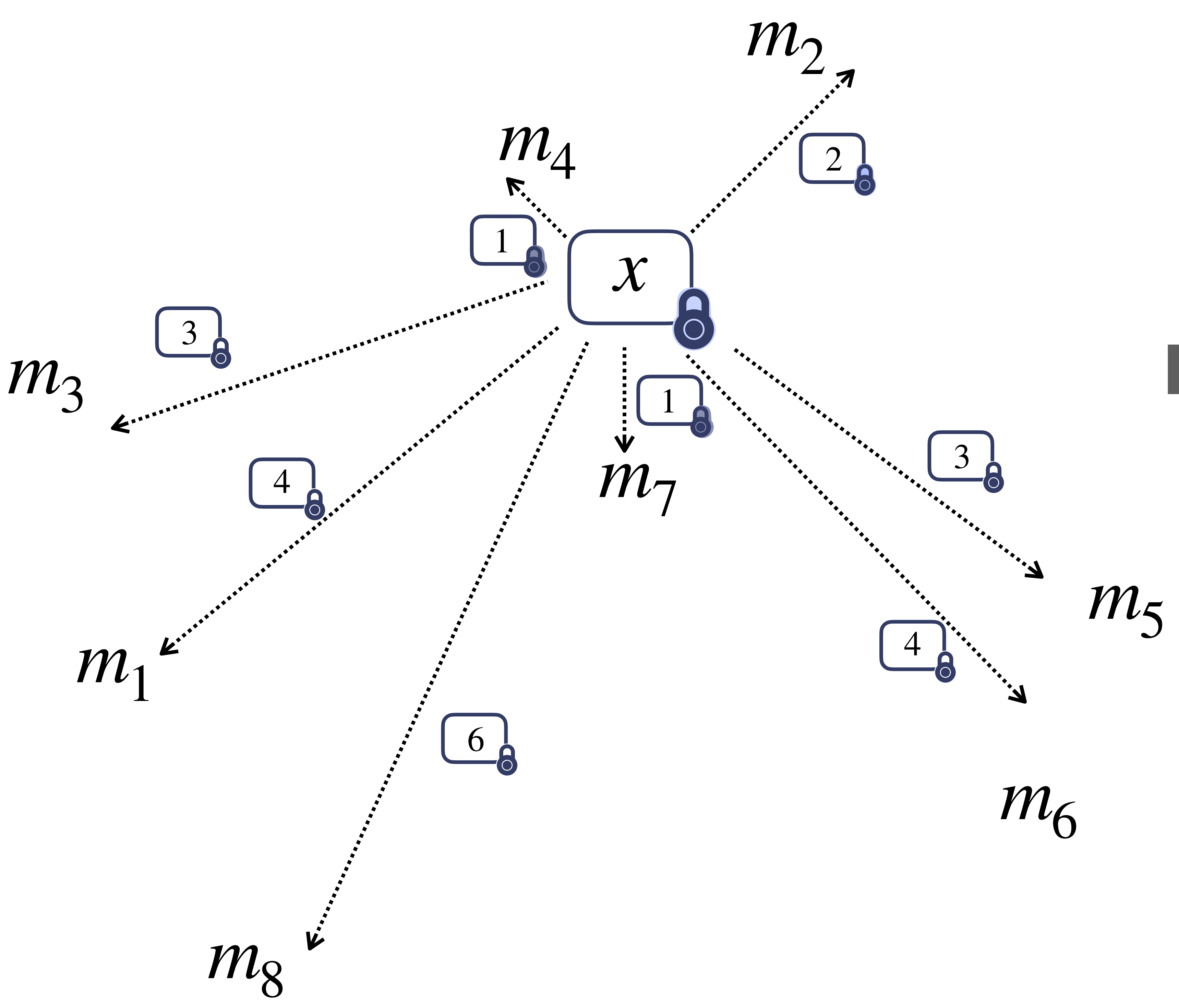

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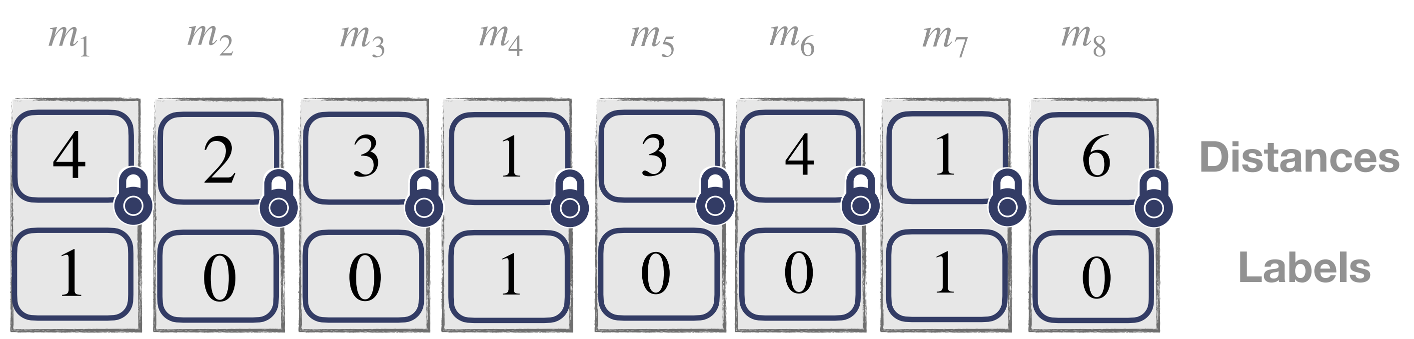

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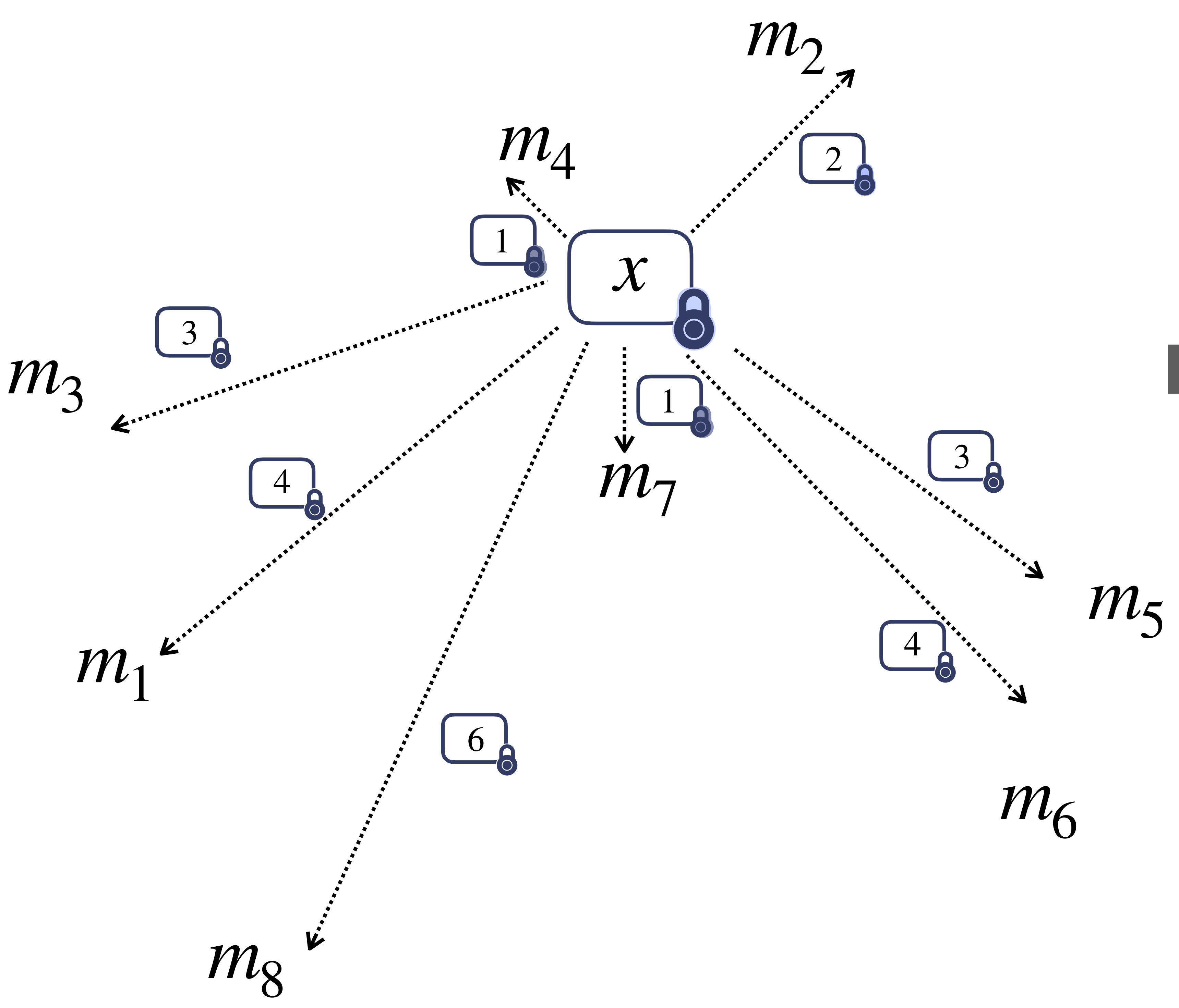

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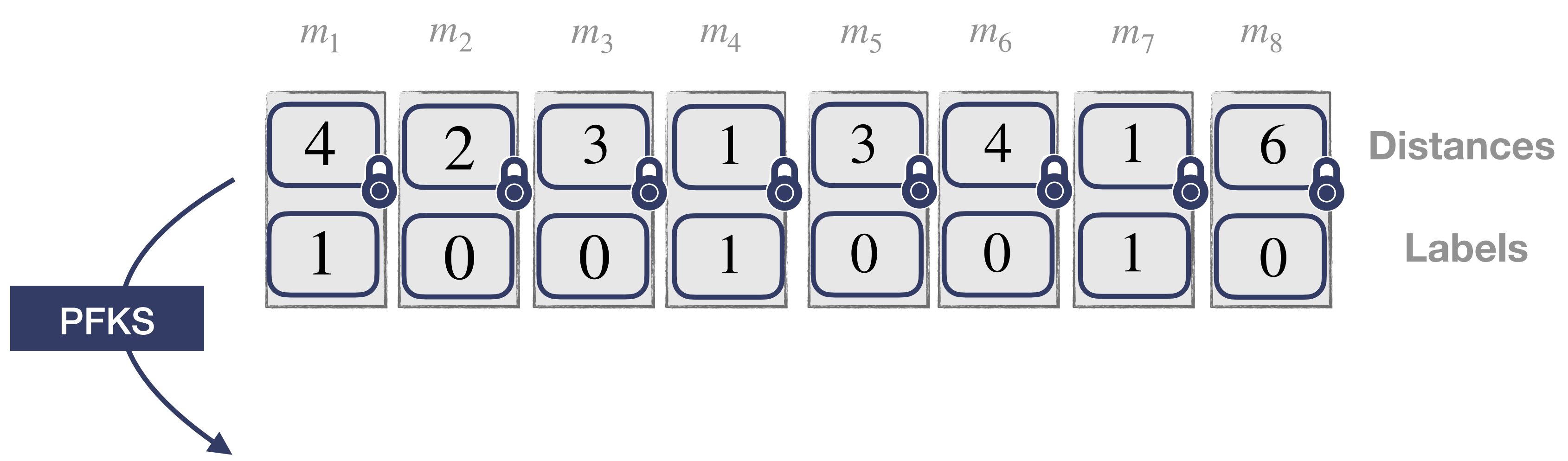

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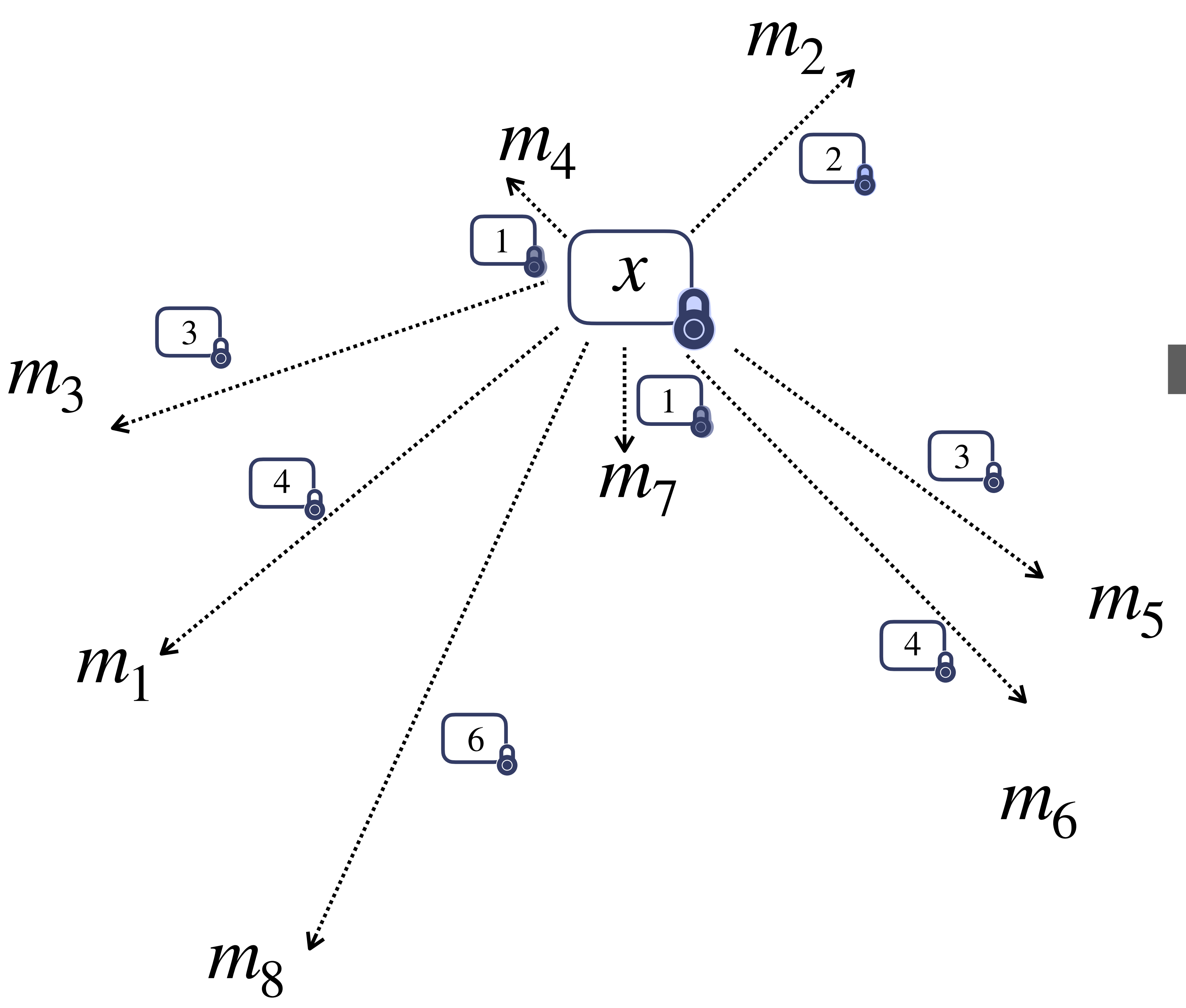

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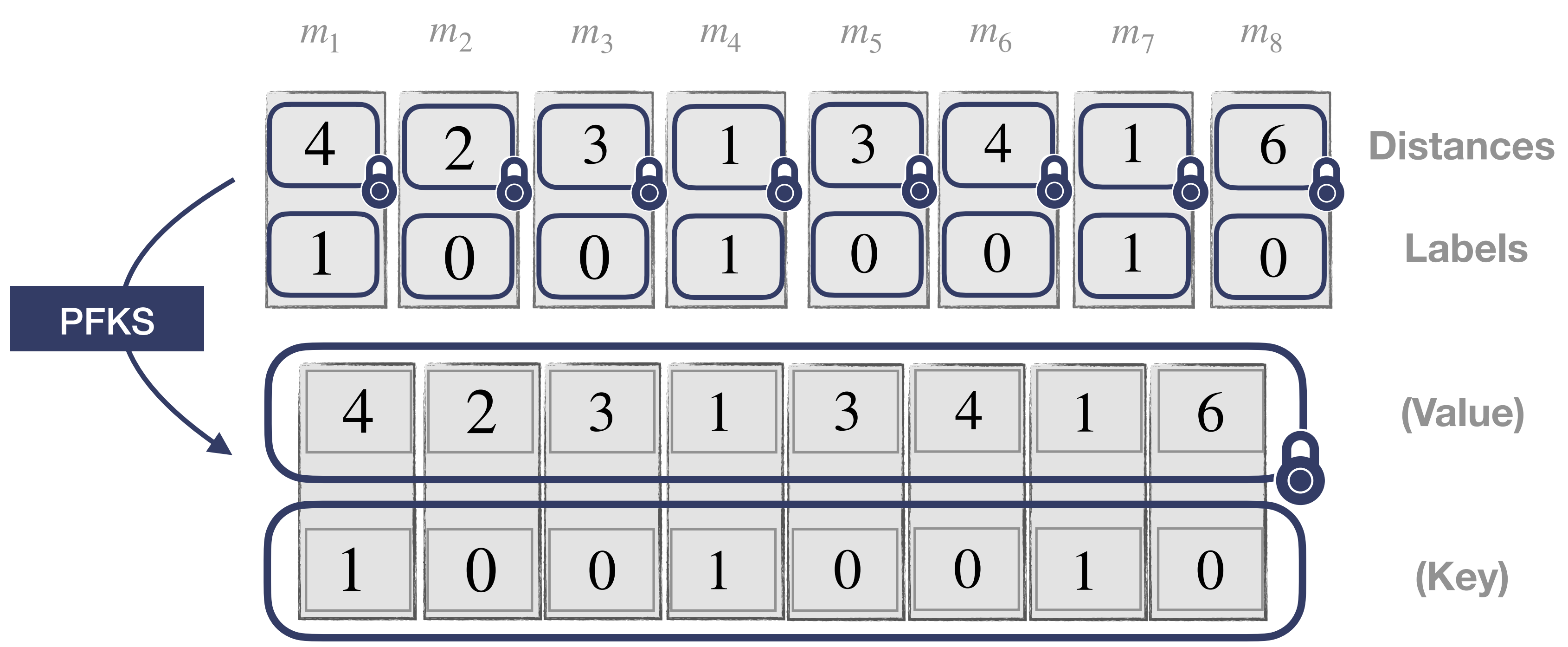

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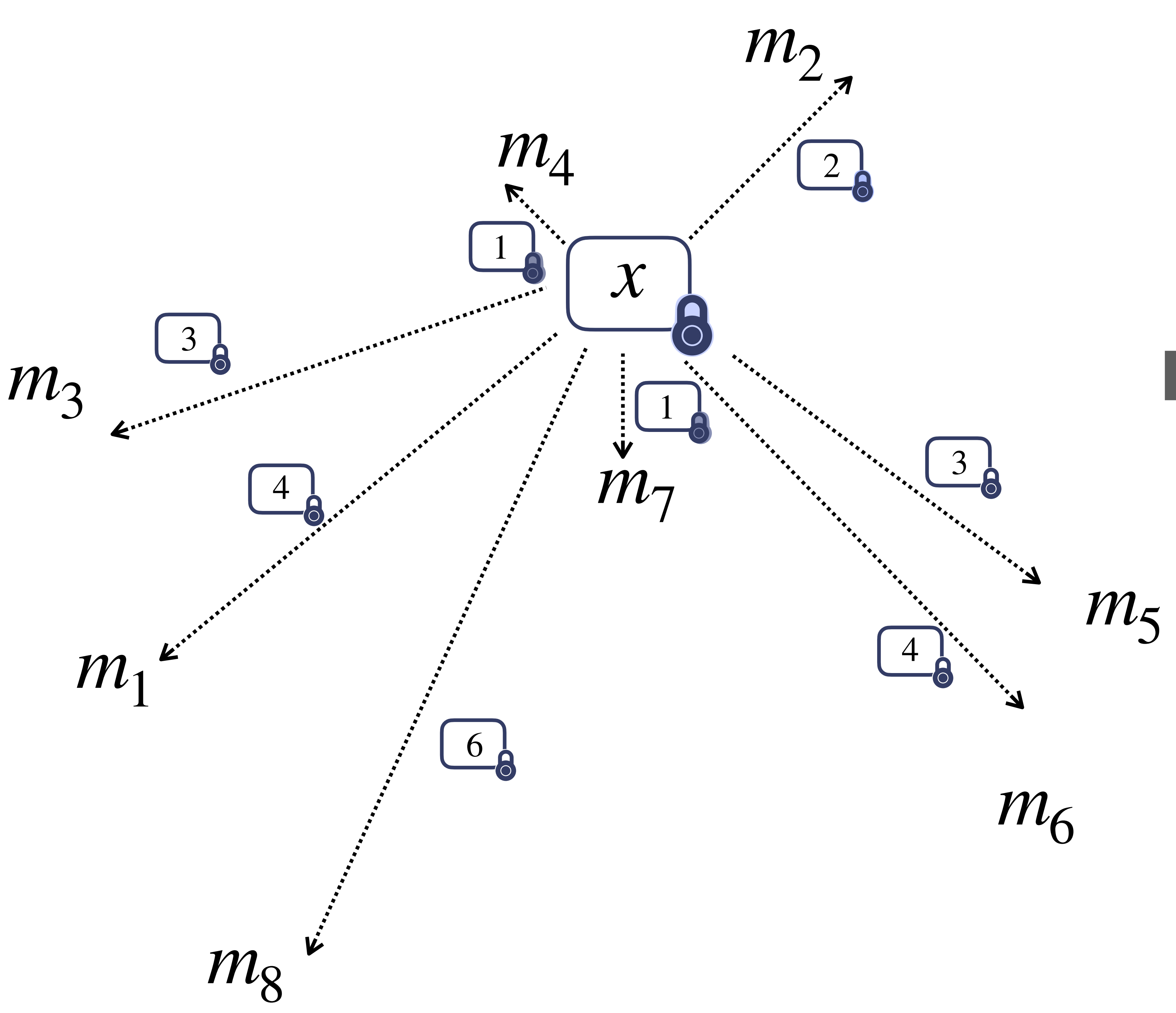

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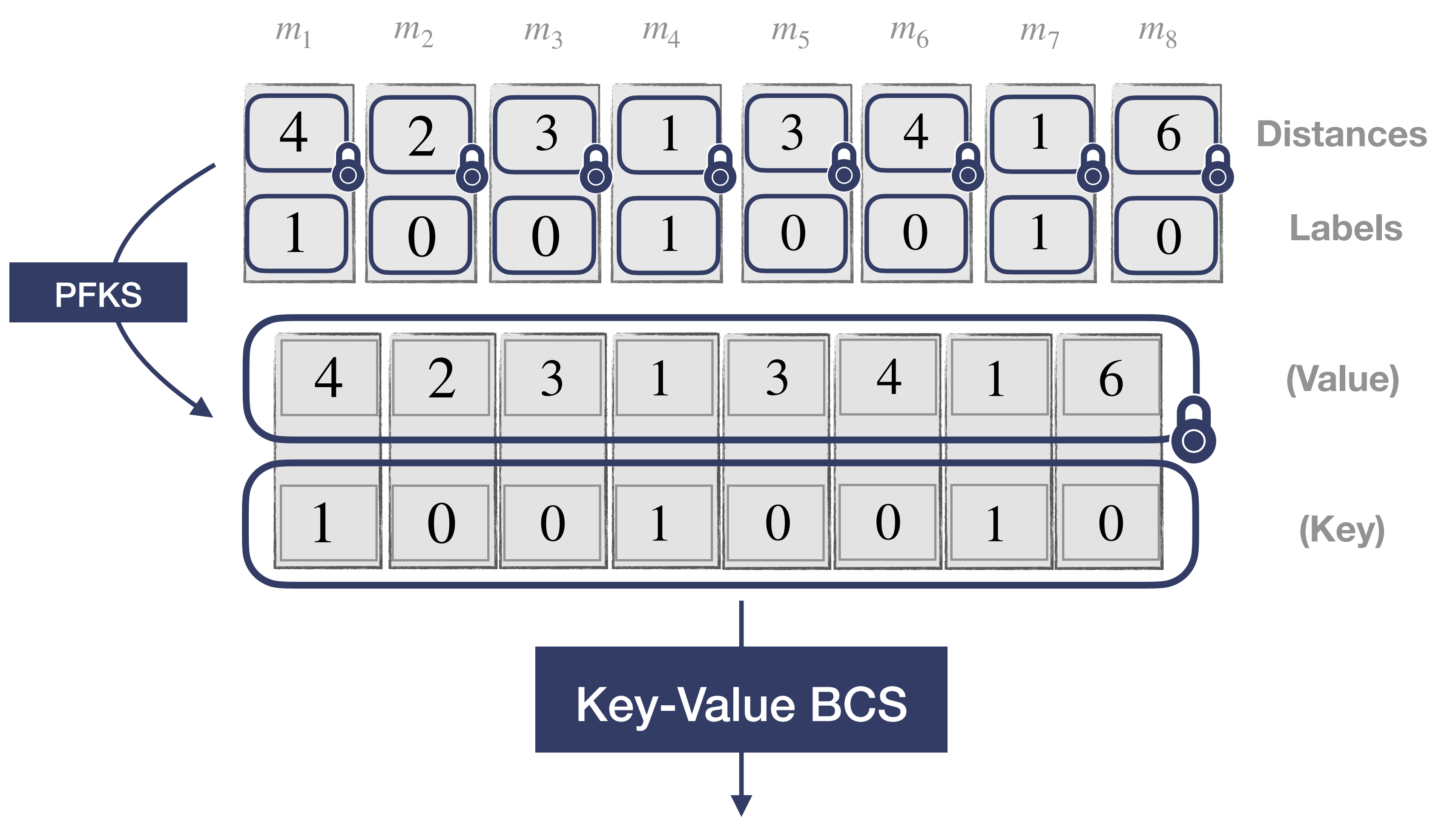

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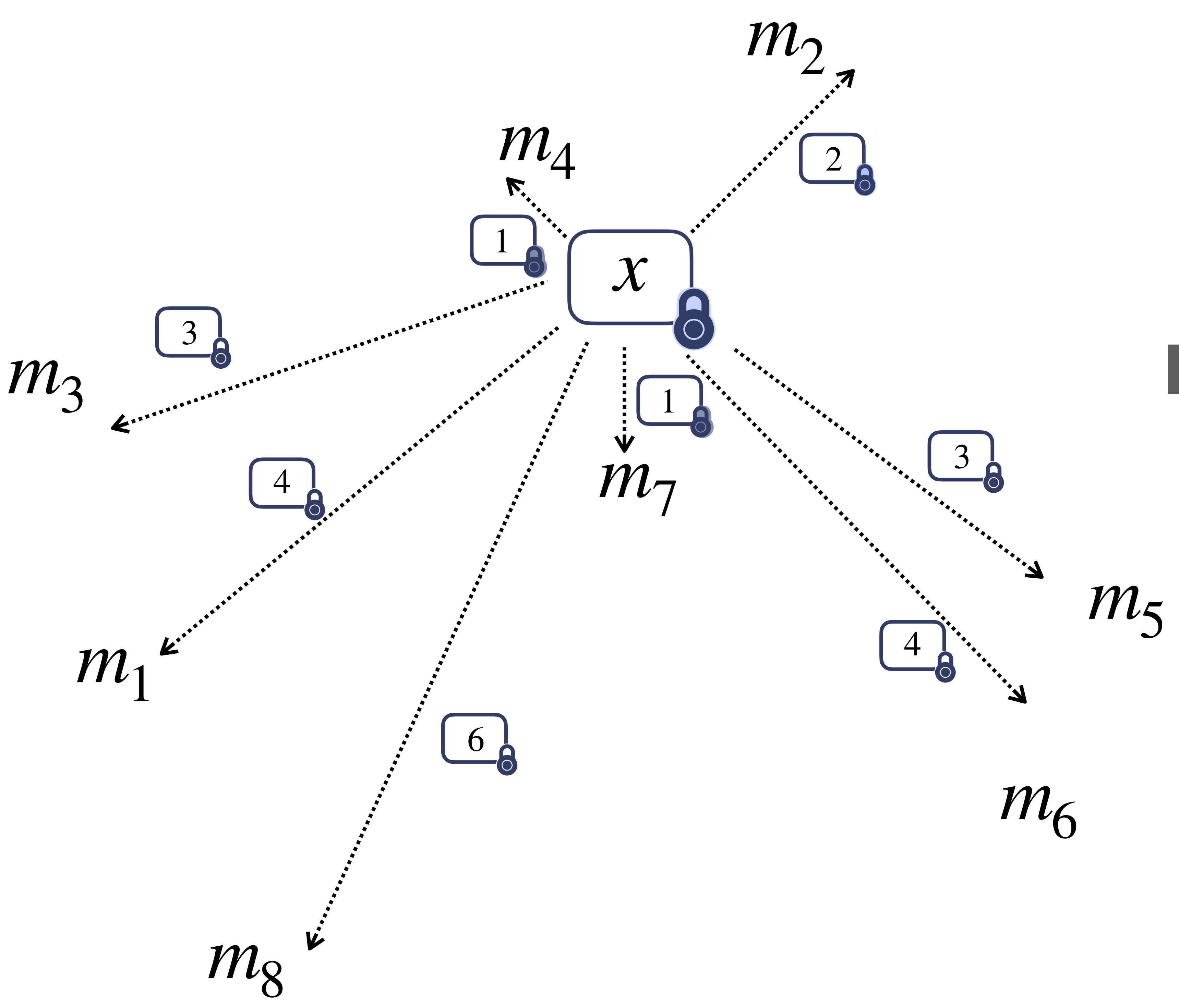

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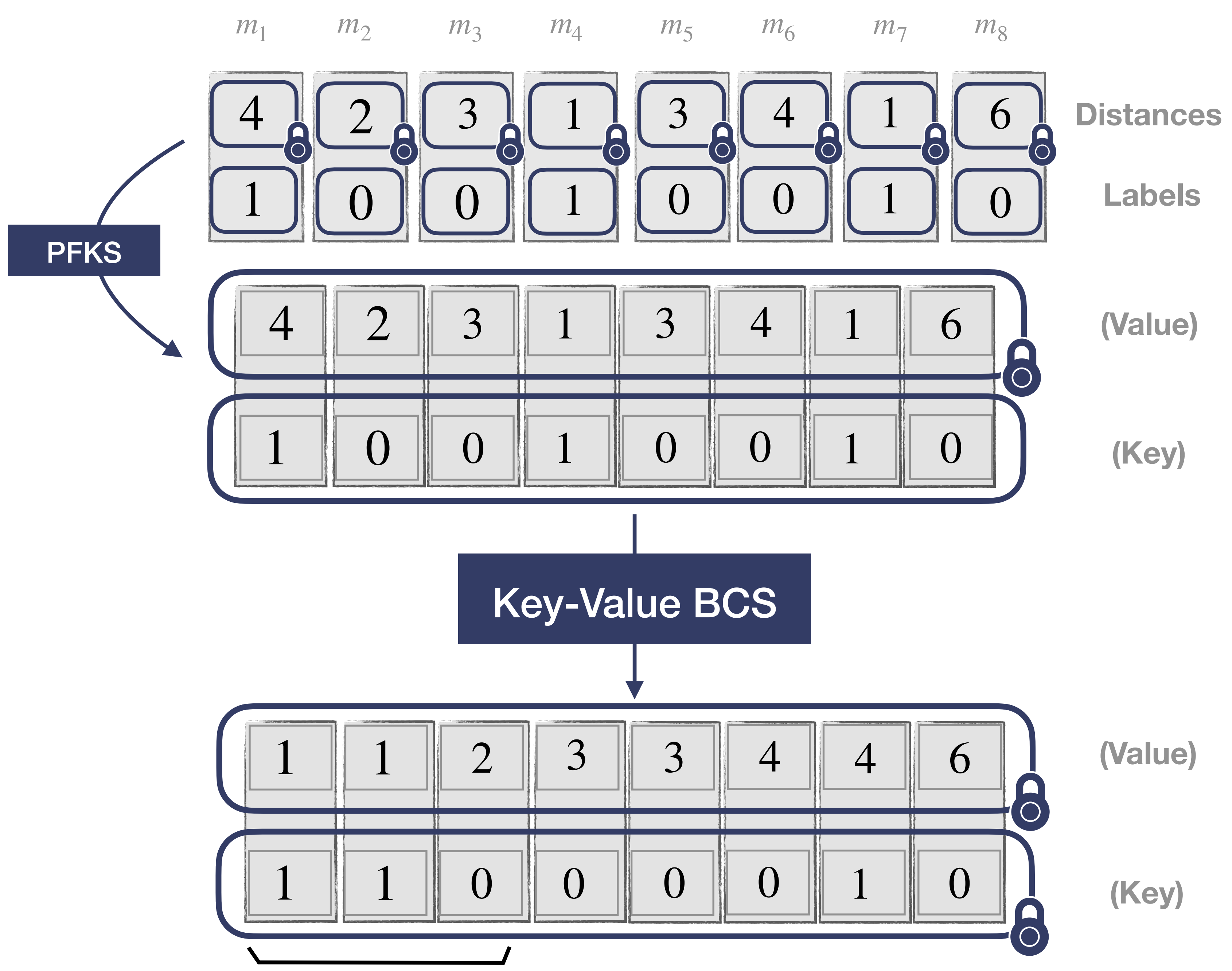

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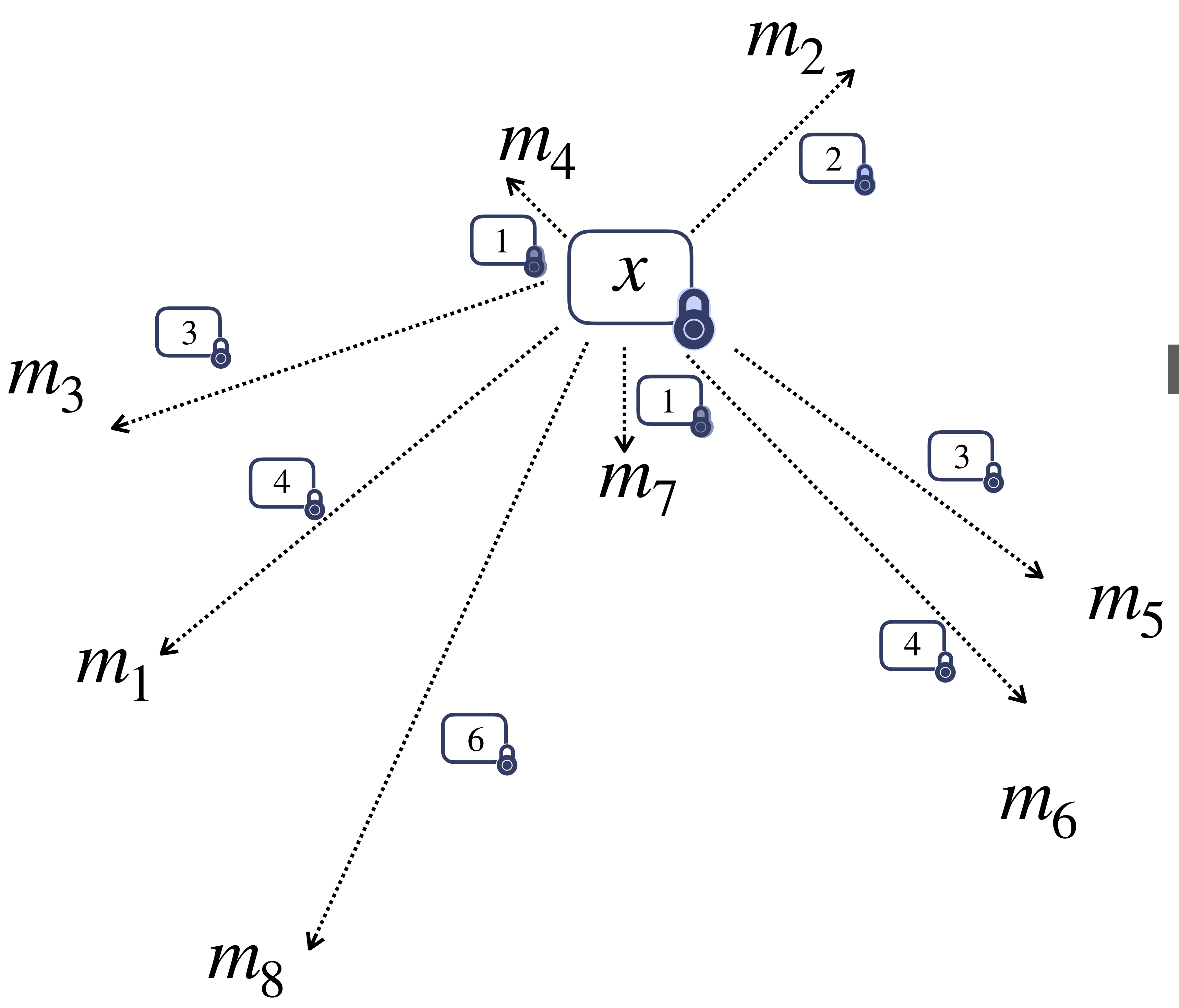

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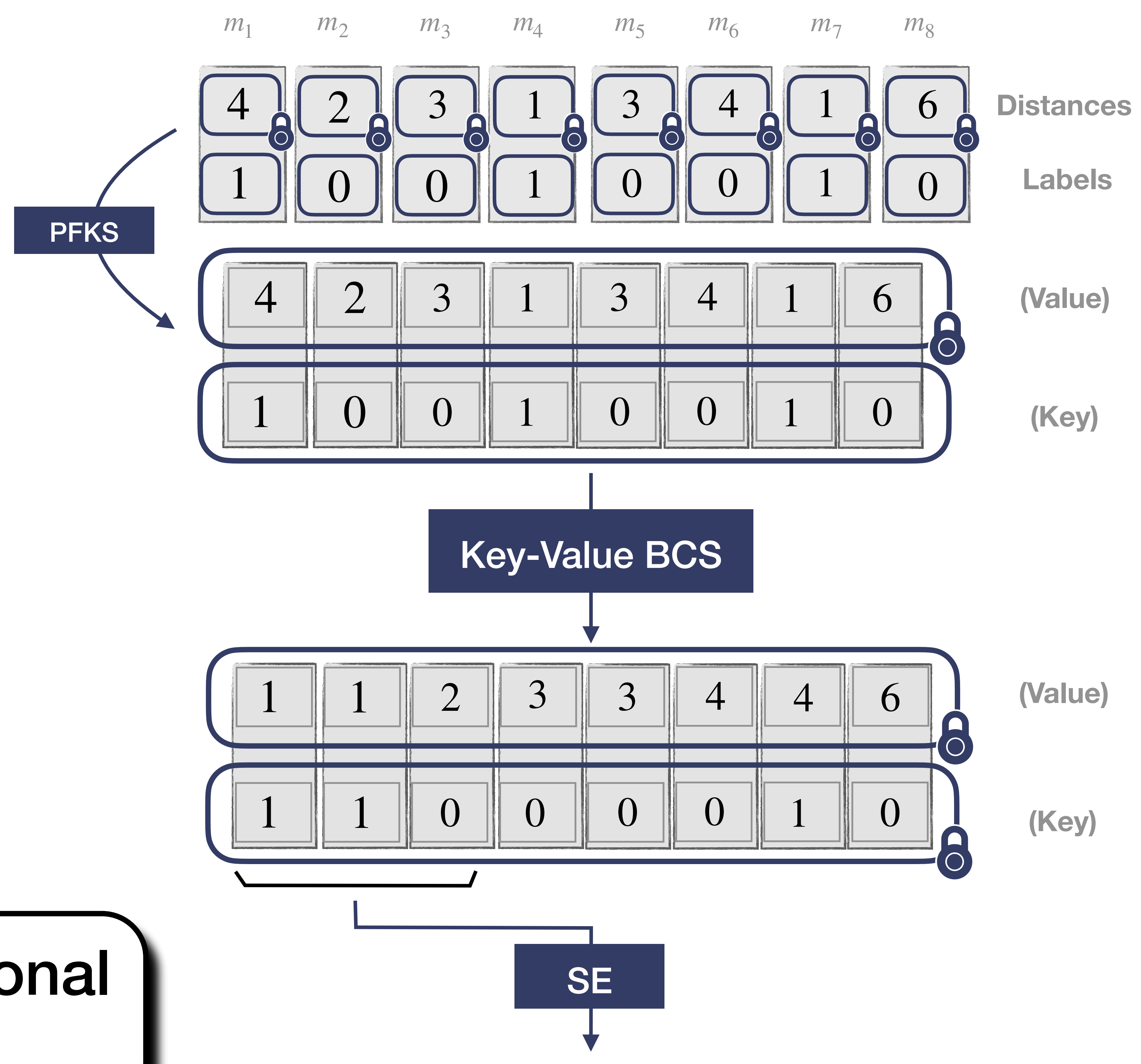

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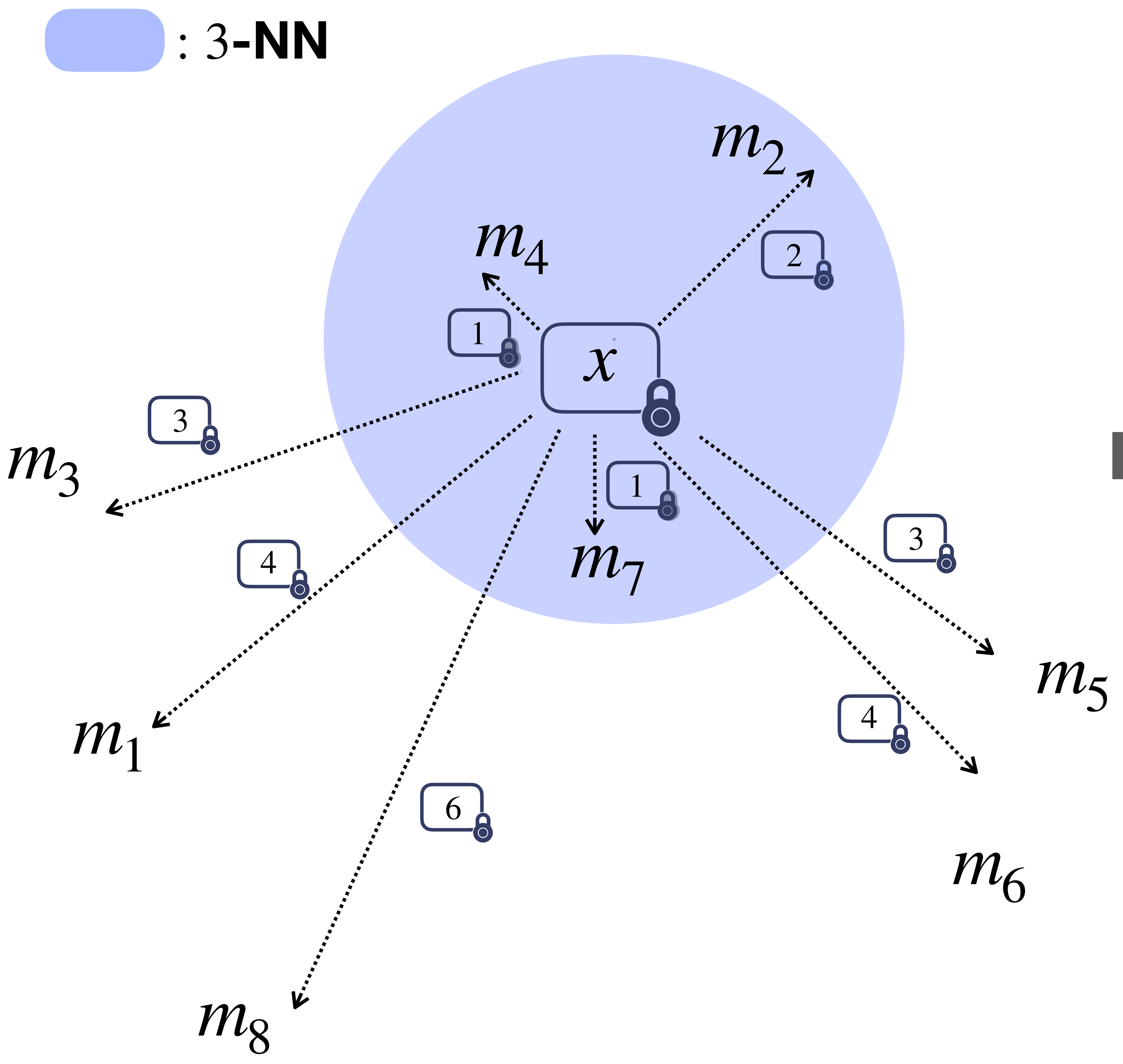

Precision reduction (if needed)

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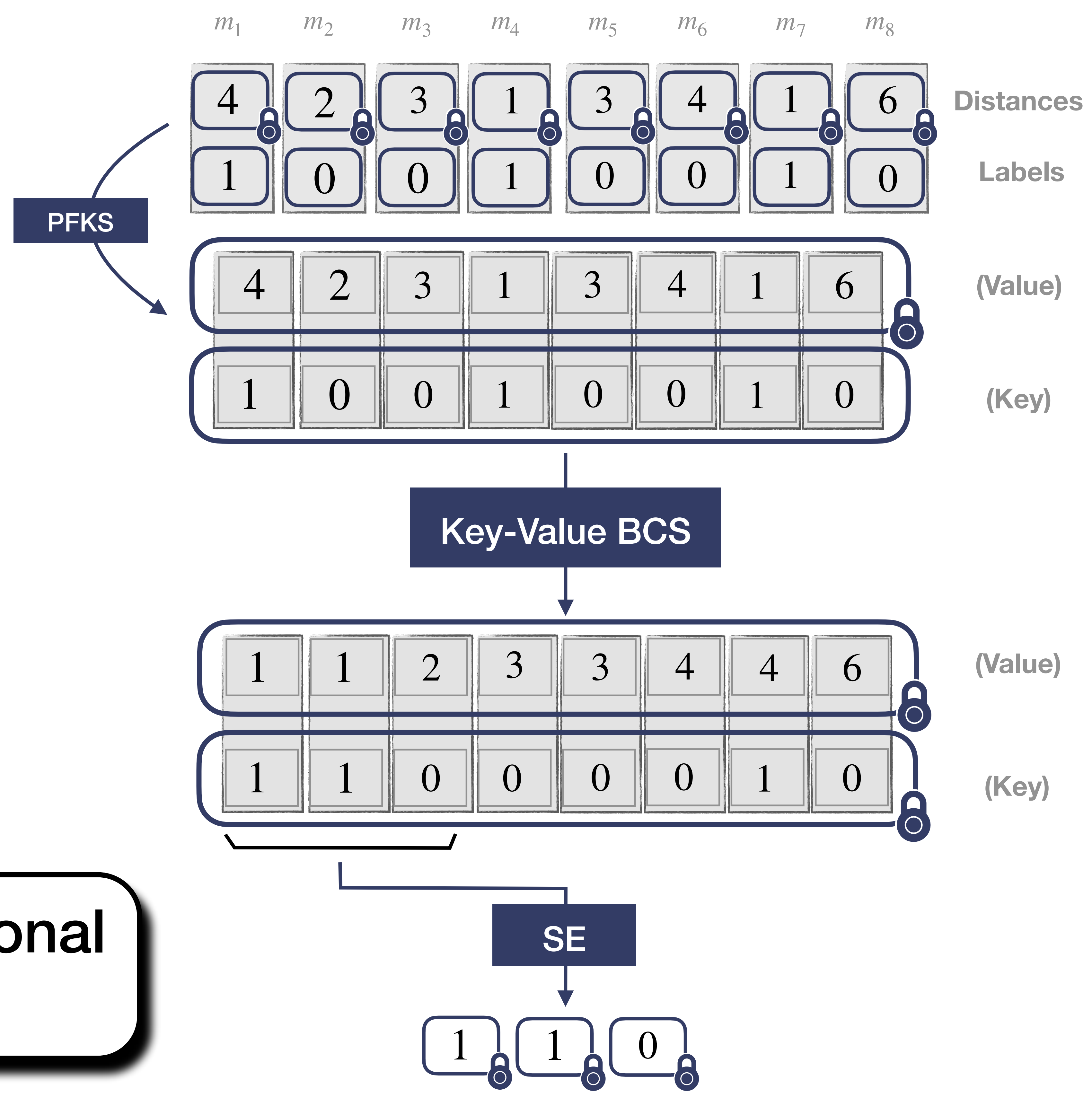
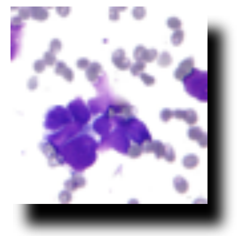
May introduce additional noise

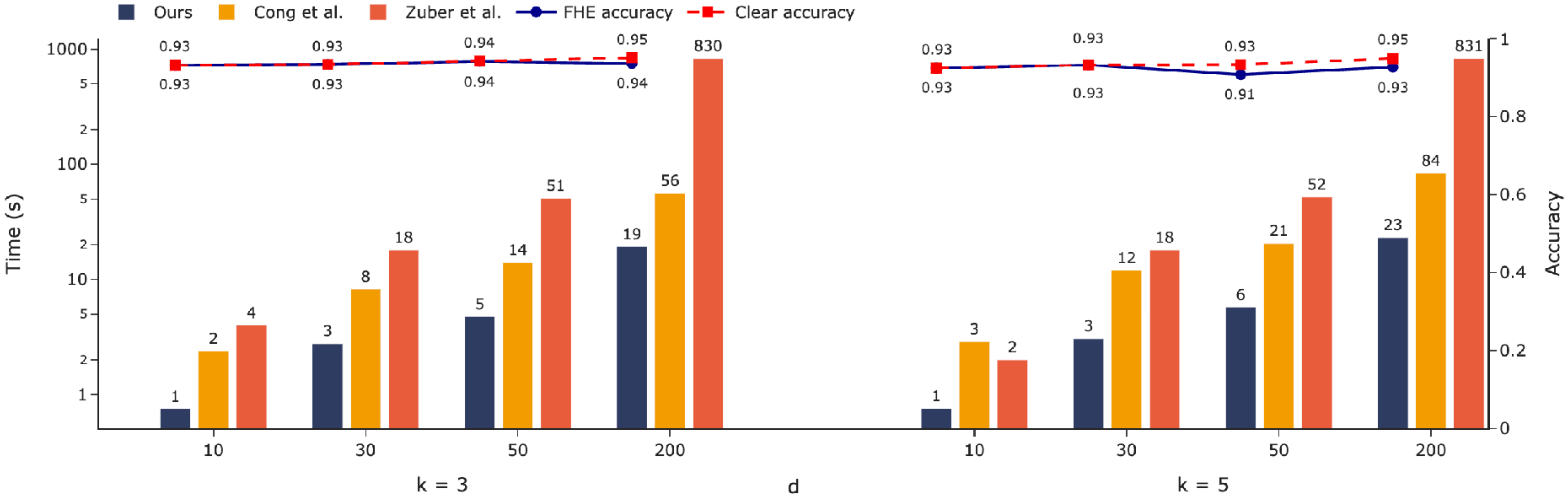
Top-k selection in $\mathbb{Z}_p$



Private k-Nearest Neighbours
Comparison with the SOA



Breast Cancer



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Private k-Nearest Neighbours
Comparison with the SOA

MNIST

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Conclusion

Conclusion

In this work, we :

- Designed the first **oblivious sorting algorithm** that operates without any comparisons
- Leveraged this sorting algorithm as a subroutine in a tournament-style to construct an **oblivious top-k** algorithm.
- Applied this oblivious top-k mechanism within a **privacy-preserving k-Nearest Neighbours model.**

Full paper: PoPETs 2025, paper no. 93

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Thank you !

Any questions ?

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